Tests and Definition of Maximum Shear Capacity of Reinforced Concrete Structures

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ABSTRACT

TESTS AND DEFINITION OF MAXIMUM SHEAR CAPACITY OF REINFORCED CONCRETE STRUCTURES

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This research was done to establish the maximum capacity of concrete in "shear", which is governing structures such as deep beams, corbels and dapped ends.

The research entailed the testing of 30 prismatic and 9 varying depth beam samples with horizontal, vertical and inclined reinforcement configurations. A high percentage of steel was used to ensure concrete failure before reaching the capacity of steel. The beams were loaded at mid point, thereby acting as a deep beam or as a twin corbel. Loading was applied in 5 kip (22 kN) increments with short time delays to observe and record the propagation of cracks. Testing revealed that nominal shear capacity significantly exceeds the current limits of both the American and Canadian codes. Test results indicate that the ratio of nominal shear stress to the compressive strength of concrete (v_u/f'_c) can be as much as 0.42 when horizontal reinforcement is used and 0.85 for inclined reinforcement, provided sufficient reinforcement is utilized. Tests also suggest that vertical stirrups do not contribute to shear strength of deep beams or corbels.

Based on these results, work models were proposed in the form of free body diagrams built of multiple compression struts and tension ties corresponding to the configuration of the reinforcement. In all cases the models rendered results with a high degree of accuracy. Ultimate shear capacity may be governed by either the compression strut (concrete) or the tension tie(steel). Maximum capacity occurs when the amount of reinforcement at least corresponds to the balanced condition, where the capacity of the tension tie is equal to or greater than that of the compression strut.

Dedicated to my mother, Irmgard, who made it all possible.

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NOTATION

a	- Shear span
A_{cv}	- Area of concrete section under shear
A_{hf}	- Area of horizontal reinforcement
A_s	- Area of reinforcement
A_{vf}	- Area of vertical reinforcement
A_{wf}	- Area of reinforcement projected on a line perpendicular to cracks
b	- Width of concrete beam or corbel
C	- Compression force
d	- Effective depth of concrete beam or corbel
D	- Dead loads
E_s	- Modulus of elasticity of steel
f'_c	- Uniaxial compressive strength of concrete (cylinder strength)
f_{cs}	- Reduced concrete compressive strength
f_{ct}	- Compression stress in concrete under biaxial tension and compression loading
f_s	- Stress in steel
f_{to}	- Uniaxial tensile strength of concrete
f_{tc}	- Tension stress in concrete under biaxial tension and compression loading
f_{tt}	- Biaxial tensile strength of concrete
h	- Height of concrete beam or corbel
L	- Live load

T - Tension force

T_h - Horizontal component of tension force

T_v - Vertical component of tension force

T_y - Tensile capacity provide by steel

v - Nominal shear stress (V/bh)

V - Factored shear load

V_{cr} - Calculated shear cracking strength

v_{icr} - Shear stress at the time of inclined cracking

V_{icr} - Shear force at the time of inclined cracking

V_n - Shear strength

V_r - Factored shear resistance

V_{su} - Ultimate strength in pure shear in terms of stress in concrete

V_u - Ultimate shear force

v_u - Ultimate nominal shear stress (V_u/bh)

V_{uc} - Ultimate shear strength based on concrete capacity

V_{ucr} - Ultimate shear cracking force

V_{up} - Ultimate shear strength with proposed limited shear capacity f_y=40000 psi

V_{us} - Ultimate shear strength based on reinforcement capacity

z - Lever arm between horizontal components of tension and concrete capacities

GREEK

α - Angle between reinforcement and a line perpendicular to cracks

ε_{ct} - Limit strain in concrete in tension direction under biaxial compression-tension

ε_{ctu} - Proposed maximum usable steel strain

ϕ - Diameter of reinforcement

ϕ_c - Concrete capacity reduction factor = 0.6 (Canadian code)

ϕ_s - Steel capacity reduction factor = 0.85 (Canadian code)

ϕ_{shear} - Shear reduction factor = 0.85 (American code)

μ - Shear friction coefficient

ρ_{hf} - Horizontal reinforcement ratio (A_{hf}/bh)

ρ_{vf} - Vertical reinforcement ratio (A_{vf}/bh)

ρ_{wf} - Projected or inclined reinforcement ration (A_{wf}/bh)

σ_c -Principal compression stress in concrete under biaxial stress state

σ_t -Principal tension stress in concrete under biaxial stress state

CHAPTER I
INTRODUCTION

1.1 General

A better understanding of the behavior of reinforced concrete is essential in establishing codes and standards. There exists a degree of ambiguity surrounding the application of existing strength limits to structures such as corbels, deep beams, wall-beams and beams with dapped ends which are subjected to a predominant shear force. This research, (conducted at Concordia University), was designed to enhance the understanding of the maximum capacity of concrete in shear.

The use of free body diagrams is an accepted practice in the analysis of concrete structures. However, the use of free diagrams to analyze shear behavior has not been given its due consideration. After cracking, concrete primarily carries the compressive load, while the reinforcement carries the tensile load. When significant cracking occurs the principal compression stress trajectories in the concrete tend towards straight lines. Therefore the load can be approximated by a straight compression strut.

The strut-and-tie model was pioneered by Morsh[(1)]. The model comprises tension members made of steel and compression members made of concrete. This model requires stringent demands on bond. The introduction of the tied-arch model, later to be known as the compression strut and tension tie model, showed that bond was only required at the anchorage. The tied-arch model combines shear forces and moments and can lead to more economical designs such as "economy girders" by Zielinski[(2),(3)].

Numerous researchers have made significant contributions to the subject of shear strength of concrete and strut-and-tie models. Among those are Leonhardt and Walther[(4)], Mattock[(5),(6),(7)], Zielinski[(8),(9),(10)], Collins and Mitchell[(11)], Schlaich, Schafer and

Jennewein[12], Kriz and Raths[13], and others.

Although the strength work models presented in this research are defined as work models for corbels, they can be applied to any reinforced concrete element subjected predominantly to shear. Therefore the use of the word "corbel" in this document encompasses all elements such as corbels or brackets, dapped ends, and deep beams.

1.2 Scope of the Study

The purpose of this research work is to study the maximum shear strength of concrete structures, such as corbels, deep beams, dapped ends and wall beams, which are subjected to predominant shear forces. Maximum shear strength was determined by conducting laboratory tests on deep beam samples and by insuring two criteria. First, the loading was applied so that the shear span to the depth of the concrete beam was less than one ($a/h<1$). Secondly, a high percentage of steel was used to ensure concrete failure.

Based on the free body diagrams, it can be shown that by using inclined reinforcement, concrete has twice the shear capacity than when using same amount of reinforcement placed horizontally. In terms of reinforcement, inclined reinforcement is $\sqrt{2}$ times more economical than horizontal reinforcement. Both these observations are tested for their validity.

Although the present American[14] and Canadian[15] codes require that corbels be design for lateral loads as well, Mattock[16] has shown that design of a corbel for a lateral can be done separately from the vertical load. This is done by adding additional horizontal reinforcement having a yield strength equal to the tension force. The present study deals solely with vertical shear loads.

CHAPTER II

EXPERIMENTAL PROGRAM AND RESULTS

2.1 Introduction

Laboratory testing of reinforced concrete samples is an integral part of the present study. It is the only means of obtaining real data to judge the various analytical models presented.

The two test programs presented in this study were inspired by an initial study carried out in 1978 by Dr. Z. A. Zielinski. The test programs and their results are described in the following sections.

Forms were built for the production of samples using treated plywood and pine planks coated with polyurethane (to prevent water absorption and warping). A release agent was applied uniformly to the formwork. A weight ratio of 1:2:3 of high early strength Portland cement (Type III), washed sand, and gravel with a nominal size of 0.25 inches (5.1mm) was used for each mix. The water/cement ratio was altered to achieve concrete compressive strengths ranging from 1176 psi to 4846 psi (8.1 to 33.4 MPa). A sufficient amount of concrete was mixed to cast three beams and four test cylinders at any one time. Samples were cast in a flat position. The samples were vibrated and troweled to a smooth finish. After 24 hours the test beams were unmolded and left to cure in a conventional moisture cabinet with constant humidity.

2.2 Test Specimens for Program I

Prismatic beam samples measuring 38"x12"x1.75" (96.52cm x 30.48cm x 4.45cm) and reinforced with #10M bars (ϕ=0.44 in) were used in this test program. Reinforcement ratios, calculated for the full beam depth ($\rho=A_s/bh$), and the number of bars used for each series are shown in Table 2.1. There was a total of 22 samples tested. Three main reinforcement patterns were considered, these included horizontal, horizontal and vertical and inclined configurations. Test Program I is further supplemented with the data collected during the testing in 1978. At that time, six samples using an inclined reinforcement pattern and reinforced with standard #4 bars (ϕ=12.7 mm) were tested. The reinforcement provided is briefly described below and shown in Figures 2.1 to 2.6.

2.2.1 Samples with Horizontal Reinforcement

Each of the five samples in series 'S' (with eight bars uniformly distributed over the entire depth) had an equivalent reinforcement ratio of ρ_{hf}=0.059. The reinforcing details are illustrated in Figure 2.1. No stirrups were used. The three samples of series 'H', containing six bars grouped together in the tensile zone, had an equivalent reinforcement ratio of ρ_{hf}=0.044 (Fig. 2.2). No stirrups were used.

2.2.2 Samples with Horizontal and Vertical Reinforcement

Series 'C' (three samples) employed eight bars uniformly distributed along it's depth and four double leg stirrups distributed within 10 inches (25.4cm) from either end, producing equivalent reinforcement ratios of ρ_{hf}=0.059 and ρ_{vf}=0.0708 (Fig. 2.3). Series 'V' (three samples) had six bars closely grouped together in the tension zone. Five single leg stirrups were distributed vertically within 10 inches (25.4cm) from either end providing equivalent reinforcement ratios of ρ_{hf}=0.044 and ρ_{vf}=0.044 respectively (Fig. 2.4).

2.2.3 Samples with Inclined Reinforcement

Twelve (12) samples with inclined reinforcement (series 'B', 'A_{78}' and 'B_{78}') had six bars horizontally placed of which four bars were bent at 45^0 (Fig. 2.5), providing equivalent reinforcement ratios of ρ=0.044 and ρ=0.057. Two samples, $A3_{78}$ and $B3_{78}$, had only four horizontally placed bars of which three bars were bent at 45^0, providing a ratio of ρ=0.038 (Fig. 2.6). No stirrups were used. The inclined bars were placed in the shear transfer area.

2.3 Test Specimens for Program II

As a result of Test Program I, further study was deemed prudent in order to lend a more applicable dimension to the test conditions. This was achieved by altering the geometric shape of the test beams. Prismatic beam samples having the same length and width as those in program I, but of varying depth, were used in this test program. Nine samples reinforced with #10M bars (ϕ= 0.44 in) were tested. Reinforcement ratios ($\rho=A_s/bh$) and the number of bars used for each series is shown in Table 2.1. Two main reinforcement patterns were considered, these included horizontal and inclined configurations. The reinforcement provided for each test series is briefly described below and shown in Figures 2.7 to 2.9. The third sample in each series had six strain gauges glued to each of the main reinforcement bars. The strain gauges were placed along the shear interface (Fig. B.1).

As well, two prismatic beams (T1,T2) having the same geometric shape as in Test Program I were tested. Six (6) strain gauges were glued to the main reinforcement. The strain gauges were placed on each reinforcement bar along the shear interface (Fig. B.2). The reinforcement details are shown in Table 2.1 and in Figures 2.10 and 2.11.

2.3.1 Samples with Horizontal Reinforcement

Each of the three samples in series 'R' had an equivalent reinforcement ratio of ρ_{hf}=0.044. The reinforcing details are illustrated in Figure 2.7. No stirrups were used. The three samples of series 'Q', six bars grouped together in the tensile zone, had an equivalent reinforcement ratio of ρ_{hf}=0.044 (Fig. 2.8). No stirrups were used.

Sample T1 and T2 each had six horizontal reinforcement bars providing an equivalent reinforcement ratio of ρ_{hf}=0.044. Sample T1 had three pairs of reinforcement bars uniformly distributed (Fig. (2.10). Sample T2 had three pairs of reinforcement bars grouped in the tensile zone (Fig. 2.11).

2.3.2 Samples with Inclined Reinforcement

Series 'P' (three samples) had six bars horizontally placed, of which four bars were bent at 45^0 (Fig. 2.9), providing an equivalent reinforcement ratio ρ=0.044. No stirrups were used, instead the inclined bars were placed in the shear transfer area.

2.4 Testing Procedure

Test samples from Program I and II were tested as beams using the loading scheme presented in Figures. 2.12 and 2.13 respectively. In this way, each beam sample can be considered a representation of a twin corbel in an upside down loading position. The ratio of shear span to depth was less than one $(a/h < 1)$, thus ensuring a predominant shear force. The supports were provided with pin and rollers to eliminate restraints. The loading plate was provided with a pin connection. Cardboard pads were placed between the sample and the steel bearing plates to reduce the influence of surface irregularities.

Loading was performed in 5 kips (22.2 kN) increments, with short delays to allow for observations and strain gauge readings. The propagation of cracks and their patterns were recorded on the samples and documented. Photographs of the samples were taken before and after each test. Concrete cylinders of three inch diameter (ϕ=7.62 cm) and six inches (15.24 cm) in height were used to determine the concrete compressive strength (f'_c).

2.5 Test Results

Data from these tests are shown in Tables 2.2 to 2.4 and Tables 4.1 to 4.10 with cracking patterns shown in Figures A.1 to A.30. All test results are referred to by the load as seen by one support. However, the photographs are labeled with the total load applied by the Tinius Olson compression machine. The values described immediately below are average values determined from collected data for each series. Data from the strain gauges is presented in Appendix B.

2.5.1 Horizontal Test Results

In test series 'S', flexural cracking initially appeared between the supports at 10 kips (44.5 kN). The average length of the initial cracks were 1/3 of the beam depth. Inclined cracking followed closely at 11.25 kips (50.0 kN) beginning on the shear interface at the support and propagating towards the center of the loading plate. The average nominal shear stress at failure, v_u, was found to be 1814.6 psi (12.5 MPa). The ratio of average nominal shear stress at failure to the compressive strength of concrete, v_u/f'_c, was 0.44. Sample S2 was spoiled and excluded from the results leaving only four samples to establish the average strength.

Series 'H' showed initial flexural cracking between the bottom supports at 11.65 kips (51.8 kN) and inclined cracking (similar to series 'S') at 15.3 kips (68.3 kN). The average length of cracks was 3/4 of the beam depth. Series 'H' yielded higher results with an average v_u=2290.5 psi (15.8 MPa) and v_u/f'_c=0.68. It is important to note that the failures were abrupt and rather violent Series 'H' revealed fewer cracks than in series 'S'.

Series 'R' displayed initial flexural cracking between the supports at an average load of 9.2 kips (40.9 kN). Inclined cracking followed at 15.8 kips (70.4 kN). The average nominal shear stress at failure was v_u=1412.7 psi (9.7 MPa) with a ratio of v_u/f'_c=0.53. The cracking patterns were similar to those observed in series 'S'. Data from the strain gauges placed on sample R3 are shown in Table B.1 and in Figure B.3.

Series 'Q' showed initial flexural cracking at 10.8 kips (48.2 kN) between the two supports followed closely by inclined cracking at 11.7 kips (52.3 kN). Inclined cracking began at the supports and propagated towards the loading plate. Tests showed an average nominal shear stress at failure v_u=1566.6 psi (10.8 kN) with a ratio of v_u/f'_c=0.48. Data from the strain gauges placed on sample Q3 are presented in Table B.2 and Figure B.4.

Sample 'T1' showed inclined cracking at 12.5 kips (55.6 kN). No flexural cracks were noticed. The nominal shear stress at failure was v_u=654.8 psi (4.51 MPa) and the ratio of nominal shear stress to compressive strength of concrete, v_u/f'_c=0.56. Sample 'T2' showed inclined cracking at 11.25 kips (50.0 kN) with flexural cracking at 16.25 kips (72.3 kN). Nominal shear stress at failure was v_u=773.8 psi (5.33 MPa) with v_u/f'_c=0.66. Data from the strain gauges for series 'T' are shown in Tables B.3, B.4 and in Figures B.5 and B.6.

2.5.2 Horizontal and Vertical Reinforced Test Samples

Series 'C' showed flexural cracking at 5.4 kips (24 kN) between the two bottom supports. Inclined cracking began at 12.5 kips (55.6 kN) at the center of the bottom supports and continued towards the shear interface. Failure occurred along the shear interface. The results indicated an average failure stress v_u=1605.5 psi (11.1 MPa) and a ratio of v_u/f'_c=0.42.

Series 'V' showed signs of flexural cracking at 11.6 kips (51.8 kN) and inclined cracking at 15 kips (66.7 kN). This series had fewer cracks than series 'C'. An average failure stress of v_u=2426.0 psi (16.7 MPa) with a ratio of v_u/f'_c=0.67 was observed. It should be noted that sample series 'C' and 'V' behaved very similar to series 'S' and 'H' respectively. From this it could be concluded that vertical stirrups do not contribute to the shear capacity of concrete corbels.

2.5.3 Inclined Reinforced Test Samples

In series 'B', samples B1 and B3 were spoiled. Samples B4 and B5 did not fail, exceeding the capacity of the loading machine. Although their maximum shear capacities are lower than their ultimate capacity, they are included in calculating average values.

Flexural cracking initially occurred at 12.9 kips (57.4 kN). Inclined cracking began at 15.8 kips (70.5 kN) beginning at the shear interface leading to the center of the loading plate. Results suggest an average v_u=2280.6 psi (15.7 MPa) and v_u/f'_c=0.69.

In series 'A_{78}', it was not possible to determine the average load at which the initial flexural crack occurred. However, the inclined cracking occurred at 30.7 kips (136.7 kN). Tests showed the nominal shear stress at failure to be v_u=1726.0 psi (11.9 MPa) with v_u/f'_c=0.80. Sample $A1_{78}$ was spoiled and excluded.

Series 'B_{78}' indicated initial flexural cracking at 29.2 kips (129.8 kN) and inclined cracking at 20.8 kips (92.8 kN). The average nominal shear stress at failure was determined to be 2140.0 psi (14.7 MPa) and the ratio v_u/f'_c=0.44. It should be noted that beams of this series failed due to steel capacity while the concrete capacity was not reached. The inclined reinforced samples of Test Program I had greater ultimate strengths, in terms of shear stress to concrete capacity v_u/f'_c than those samples with horizontal reinforcement.

Series 'P' displayed initial flexural cracking at 11.7 kips (51.8 MPa) between the bottom supports. Inclined cracking occurred at 15.0 kips (66.7 MPa) starting three (3) inches (7.62 cm) from the edge of the support and propagating to the end of the hook anchor for the inclined reinforcement bar. The cracking pattern of this series was quite different from all the others. Large cracks occurred quickly along the perimeter of the test beam from the support to the loading plate. Tests revealed an average failure stress of v_u=1412.7 psi and the ratio v_u/f'_c=0.49. Data from the strain gauges placed on sample P3 are shown in Table B.5 and Figure B.7.

Further inspection determined that the reduced concrete area was detrimental to this series. The inclined reinforcement introduced high tensile stresses through a reduced concrete area of high compressive stresses. This combination led to an earlier than expected failure. The ratio of v_u/f'_c was comparable to that of the tests of the horizontally reinforced samples. Due to the mode of failure it is recommended not to introduce inclined reinforcement in a varying depth corbel or deep beam.

TABLE 2.1 Details of test beams

Series	Number of samples	Number of horizontal bars	Number of vertical bars	Number of inclined bars	Reinforcement ratio ρ_{hf}	Reinforcement ratio ρ_{vf}
S	5	8	none	none	0.059	-----
H	3	6	none	none	0.044	-----
C	3	8	8	none	0.059	0.0708
V	3	6	5	none	0.044	0.044
B	8	2	none	4	0.044	-----
A_{78}	2	2	none	4	0.057	-----
$A3_{78}$	1	1		3	0.038	
B_{78}	2	2	none	4	0.057	-----
$B3_{78}$	1	1		3	0.038	
R	3	6	none	none	0.044	-----
Q	3	6	none	none	0.044	-----
P	3	2	none	4	0.044	-----
T	2	6	none	none	0.044	-----

TABLE 2.2 Test results for samples with horizontal reinforcement

Sample	f'_c psi (MPa)	f_y psi (MPa)	V_{fcr} kips (kN)	V_{lcr} kips (kN)	V_u kips (kN)
S1	3791.4 (26.1)	69000 (475.7)	7.5 (33.3)	10.0 (44.4)	35.5 (158)
S3	3791.4 (26.1)	69000 (475.7)	10.0 (44.4)	10.0 (44.4)	30.5 (136)
S4	4566.0 (31.5)	40000 (275.8)	12.5 (55.6)	12.5 (55.6)	44.5 (198)
S5	4566.0 (31.5)	40000 (275.8)	10.0 (44.4)	12.5 (55.6)	44.4 (198)
H1	3346.3 (23.1)	40000 (275.8)	10.0 (44.4)	15.0 (66.7)	50.2 (233)
H2	3346.3 (23.1)	40000 (275.8)	12.5 (55.6)	17.5 (77.8)	51.9 (231)
H3	3346.3 (23.1)	40000 (275.8)	12.5 (55.6)	13.5 (60.0)	47.3 (210)
R1	2634.0 (18.2)	60000 (414.0)	7.5 (33.3)	15.0 (66.7)	30.5 (135.6)
R2	2634.0 (18.2)	60000 (414.0)	10.0 (44.4)	15.0 (66.7)	30.0 (133.4)
R3	2634.0 (18.2)	60000 (414.0)	10.0 (44.4)	17.5 (77.8)	28.5 (126.8)
Q1	3255.5 (22.5)	60000 (414.0)	12.5 (55.6)	12.5 (55.6)	33.5 (149.0)
Q2	3255.5 (22.5)	60000 (414.0)	10.0 (44.4)	12.5 (55.6)	30.4 (135.2)
Q3	3255.5 (22.5)	60000 (414.0)	10.0 (44.4)	10.0 (44.4)	34.8 (154.8)
T1	1176.0 (8.1)	60000 (414.0)	--	12.5 (55.6)	13.8 (61.4)
T2	1176.0 (8.1)	60000 (414.0)	16.25 (72.3)	11.25 (50.0)	16.0 (71.2)

TABLE 2.3 Test results for samples with horizontal and vertical reinforcement

Sample	f'_c psi (MPa)	f_y psi (MPa)	V_{fcr} kips (kN)	V_{lcr} kips (kN)	V_u kips (kN)
C1	3346.3 (23.1)	69000 (475.7)	5.0 (22.2)	12.5 (55.6)	29.8 (132.5)
C2	3346.3 (23.1)	69000 (475.7)	6.2 (27.8)	12.5 (55.6)	35.2 (156.6)
C3	3346.3 (23.1)	69000 (475.7)	5.0 (22.2)	12.5 (55.6)	39.6 (176.4)
V1	3607.7 (24.9)	40000 (275.8)	10.0 (44.4)	10.0 (44.4)	51.5 (229.1)
V2	3607.7 (24.9)	40000 (275.8)	10.0 (44.4)	15.0 (66.7)	52.3 (232.6)
V3	3607.7 (24.9)	40000 (275.8)	15.0 (66.7)	20.0 (88.9)	54.5 (242.6)

TABLE 2.4 Test results for samples with inclined reinforcement

Sample	f'_c psi (MPa)	f_y psi (MPa)	V_{fcr} kips (kN)	V_{icr} kips (kN)	V_u kips (kN)
B2	3752.5 (25.9)	73000 (503.3)	22.5 (100.1)	17.5 (77.8)	47.5 (211.3)
B4	4329.0 (29.8)	73000 (503.3)	10.0 (44.4)	20.0 (88.9)	61.0 (271.3)
B5	4329.0 (29.8)	73000 (503.3)	10.0 (44.4)	10.0 (44.4)	61.0 (271.3)
B6	2613.0 (18.0)	40000 (275.8)	12.5 (55.6)	20.0 (88.9)	44.3 (197.0)
B7	2150.0 (14.8)	40000 (275.8)	12.5 (55.6)	15.0 (66.7)	40.0 (177.9)
B8	2150.0 (14.8)	40000 (275.8)	10.0 (44.4)	12.5 (55.6)	46.6 (207.3)
$A2_{78}$	2150.0 (14.8)	60000 (414.0)	N/A	32.5 (144.5)	38.5 (171.0)
$A3_{78}$	2150.0 (14.8)	60000 (414.0)	19.0 (84.5)	29.0 (129.0)	34.0 (151.0)
$B4_{78}$	4846.0 (33.4)	60000 (414.0)	35.0 (155.7)	25.0 (111.2)	45.0 (200.1)
$B5_{78}$	4846.0 (33.4)	60000 (414.0)	35.0 (155.7)	25.0 (111.2)	45.0 (200.1)
$B6_{78}$	4846.0 (33.4)	60000 (414.0)	17.5 (77.8)	12.5 (55.6)	45.0 (200.1)
P1	2884.8 (19.9)	60000 (414.0)	12.5 (55.6)	15.0 (66.7)	26.0 (115.6)
P2	2884.8 (19.9)	60000 (414.0)	12.5 (55.6)	15.0 (66.7)	30.5 (135.6)
P3	2884.8 (19.9)	60000 (414.0)	10.0 (44.4)	15.0 (66.7)	32.5 (144.6)

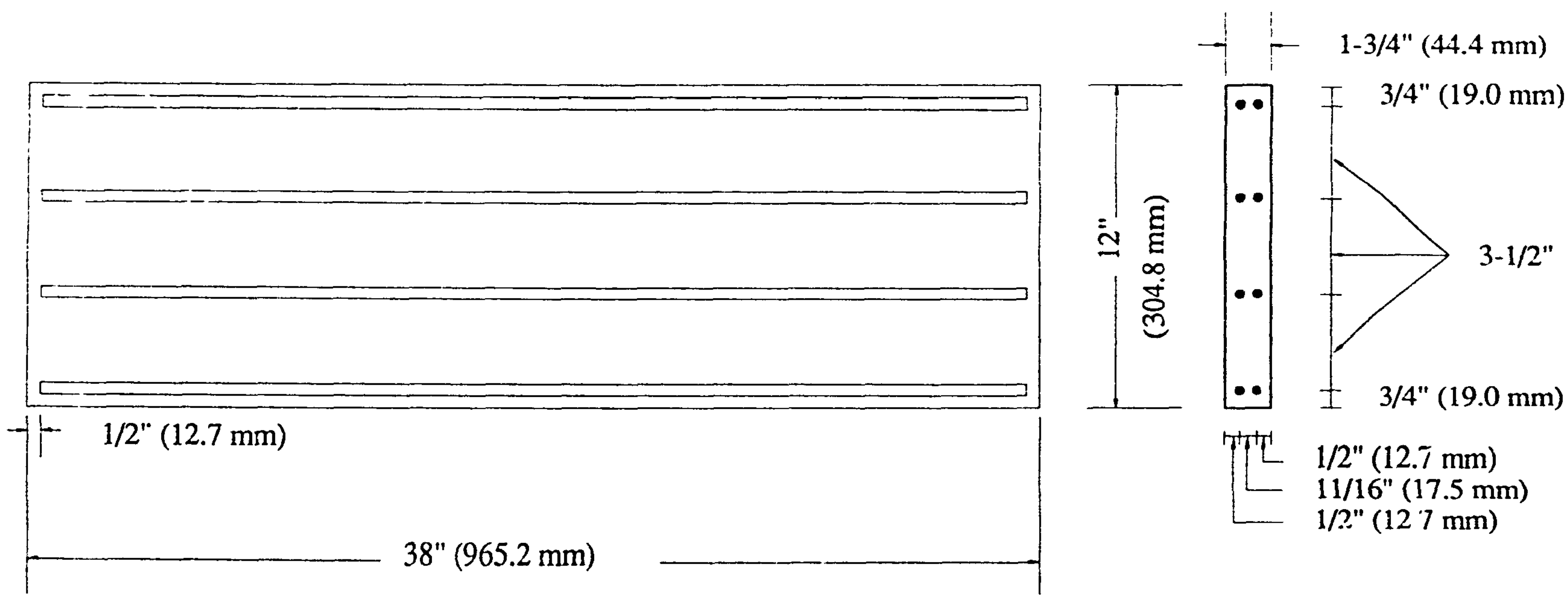


FIGURE 2.1 Reinforcement details for sample series 'S'

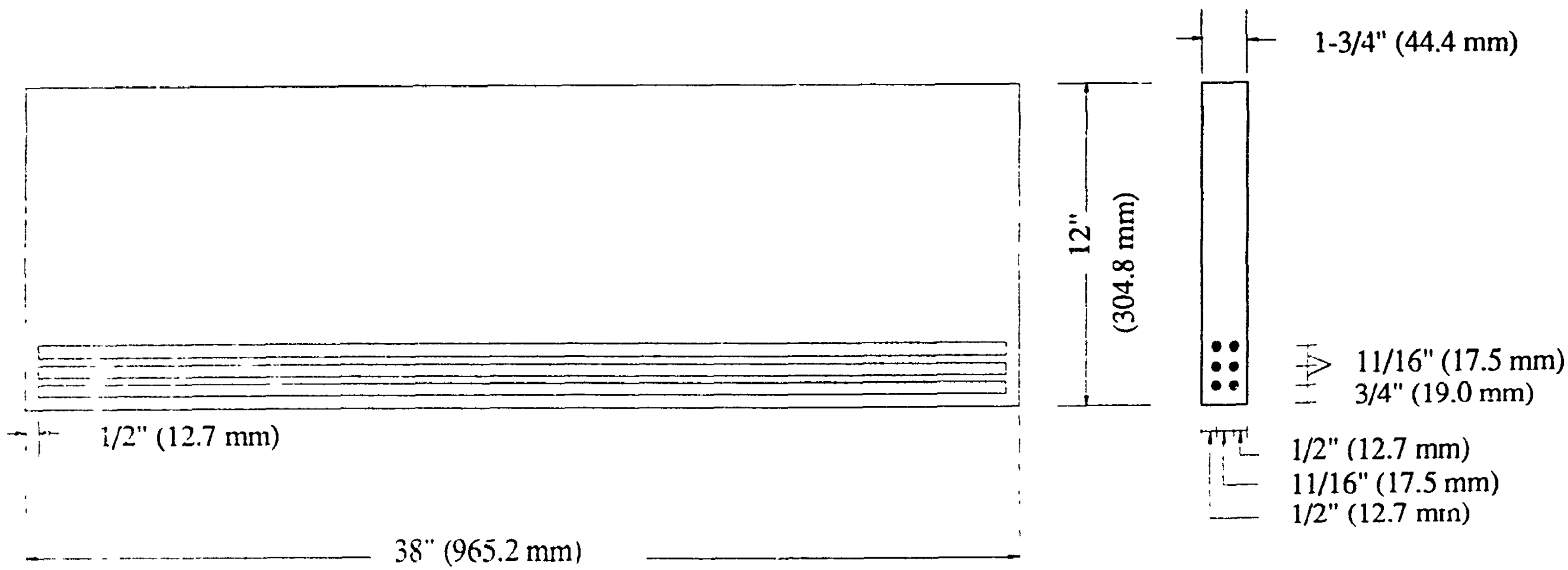


FIGURE 2.2 Reinforcement details for sample series 'H'

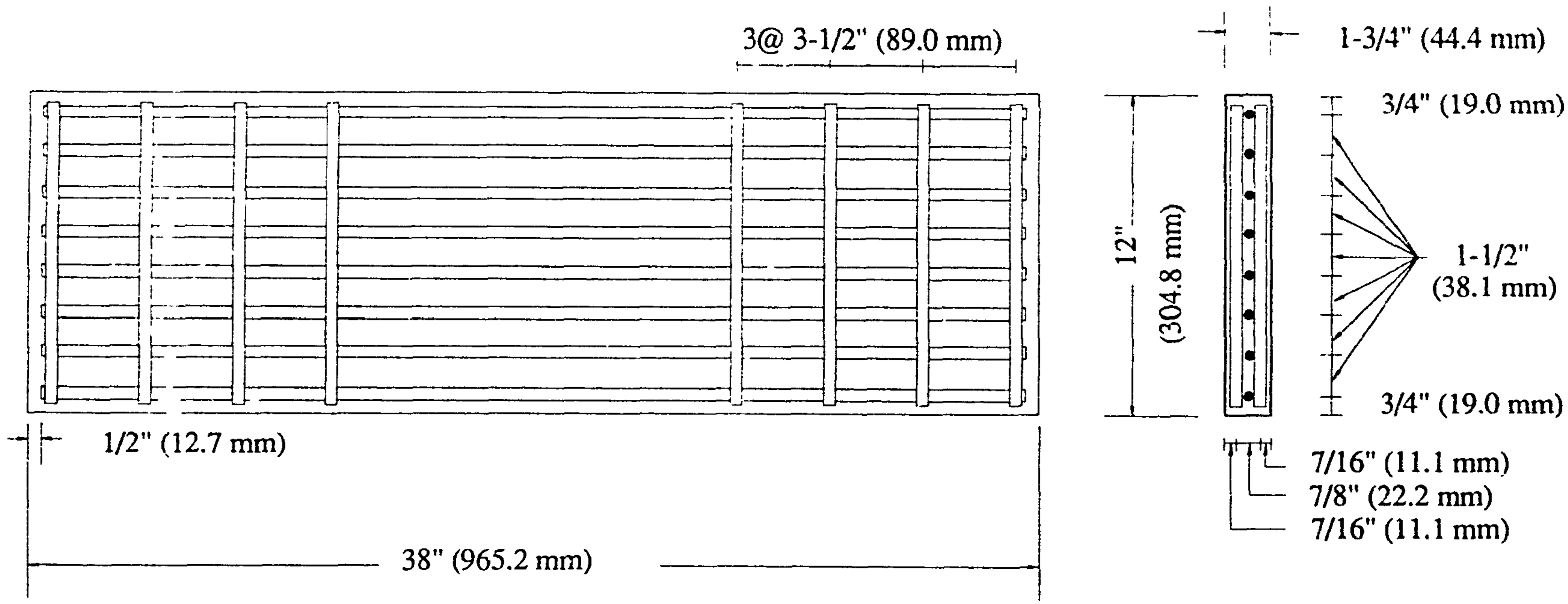


FIGURE 2.3 Reinforcement details for sample series ‘C’

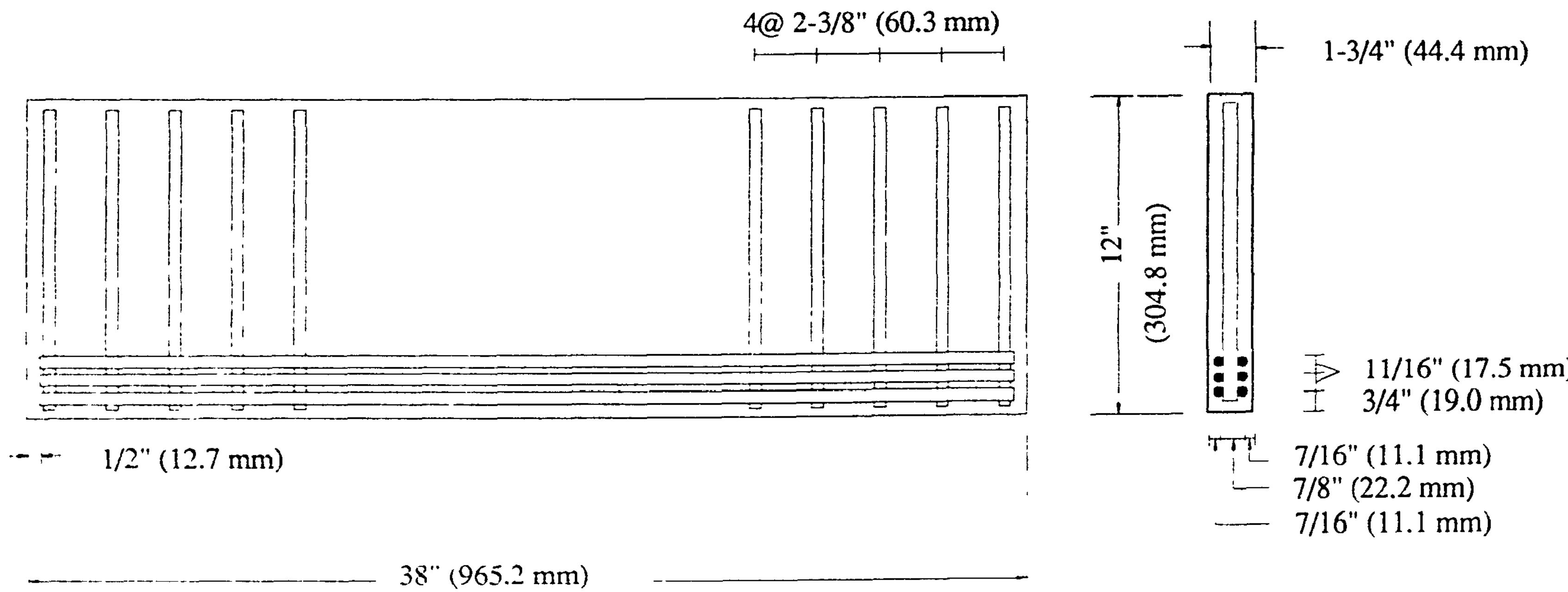


FIGURE 2.4 Reinforcement details for sample series 'V'

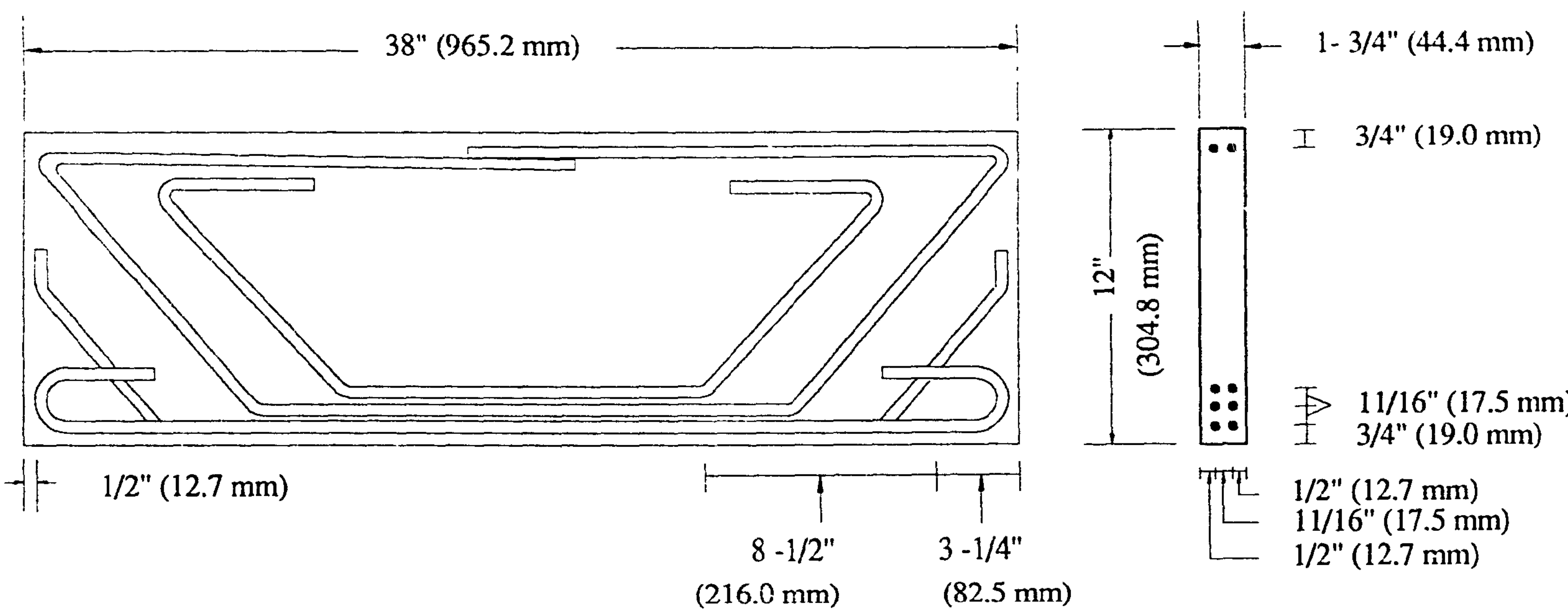


FIGURE 2.5 Reinforcement details for sample series 'B', 'A_{78}' and 'B_{78}'

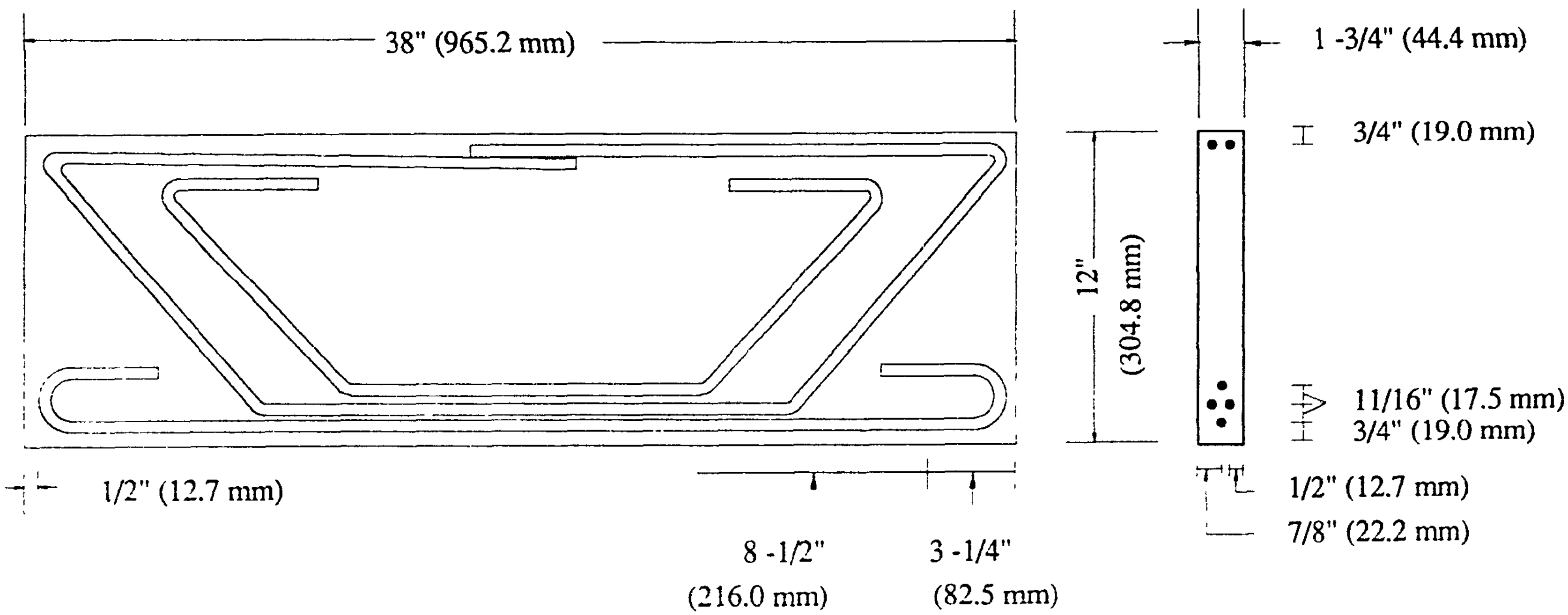


FIGURE 2.6 Reinforcement details for sample series $A3_{78}$ and $B3_{78}$

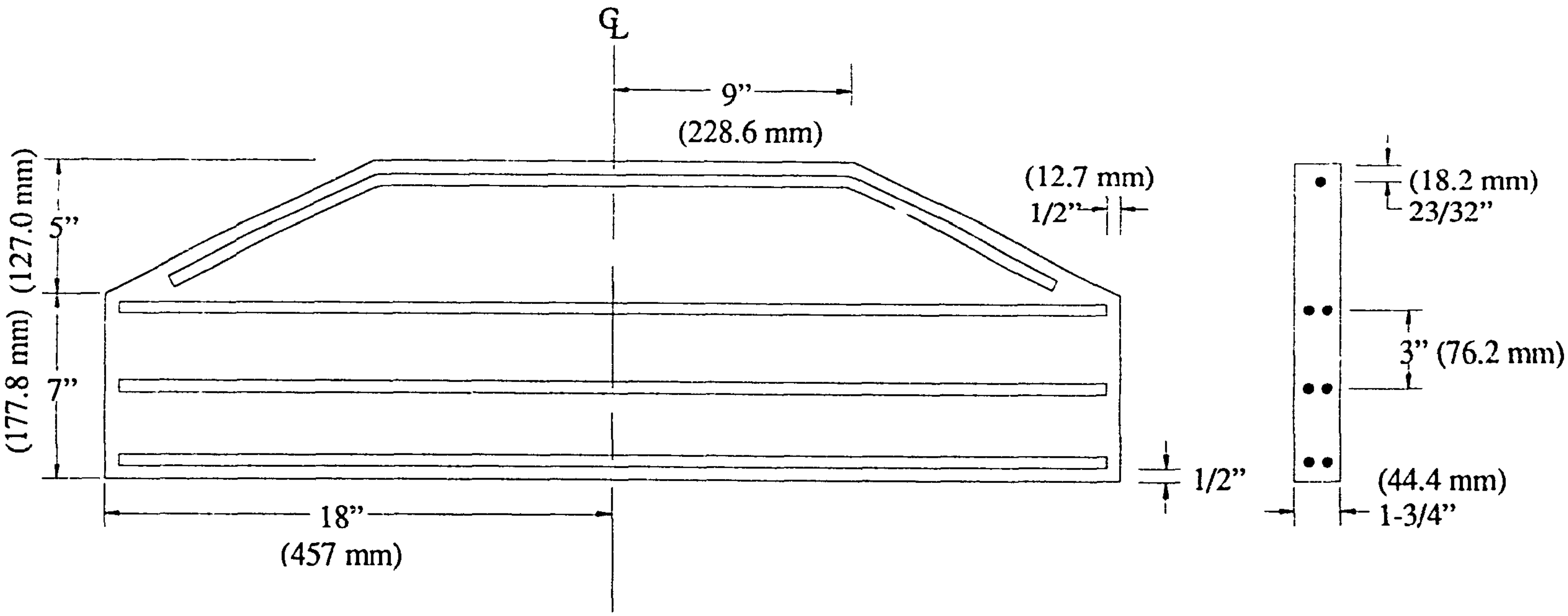


FIGURE 2.7 Reinforcement details for sample series 'R'

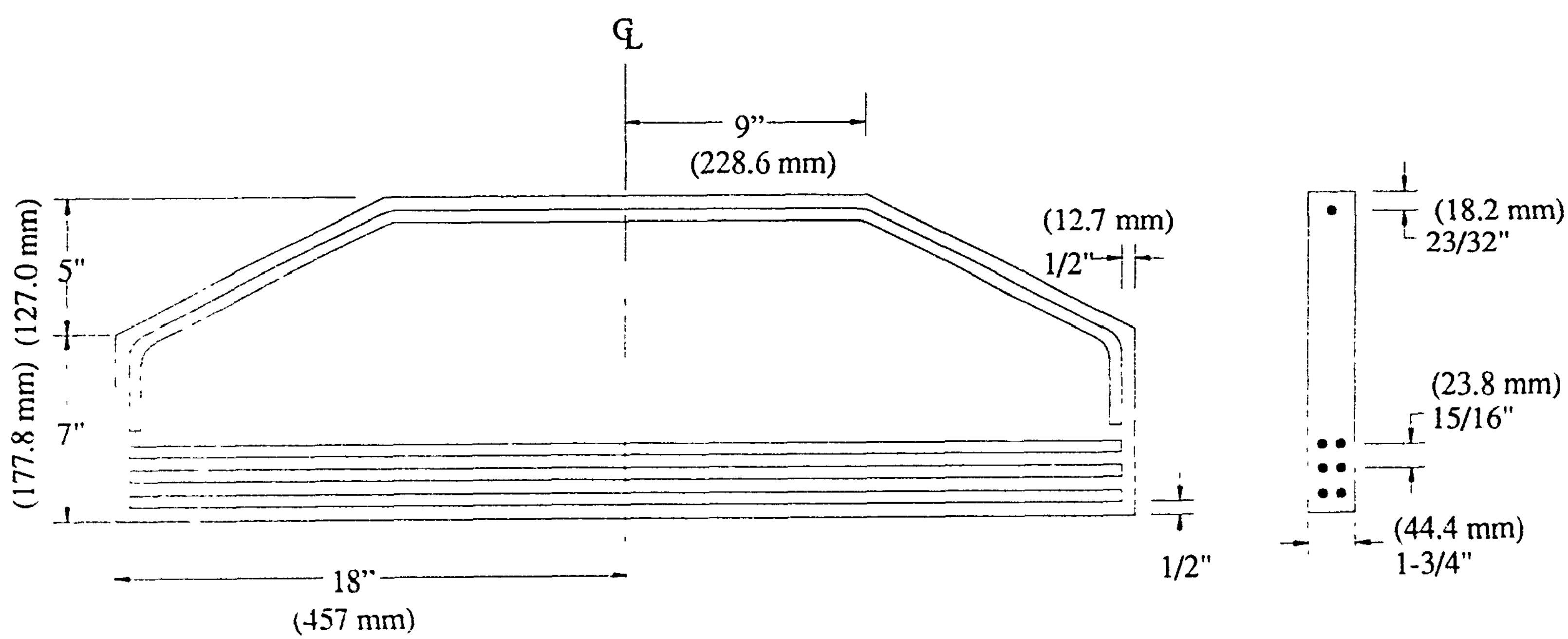


FIGURE 2.8 Reinforcement details for samples series 'Q'

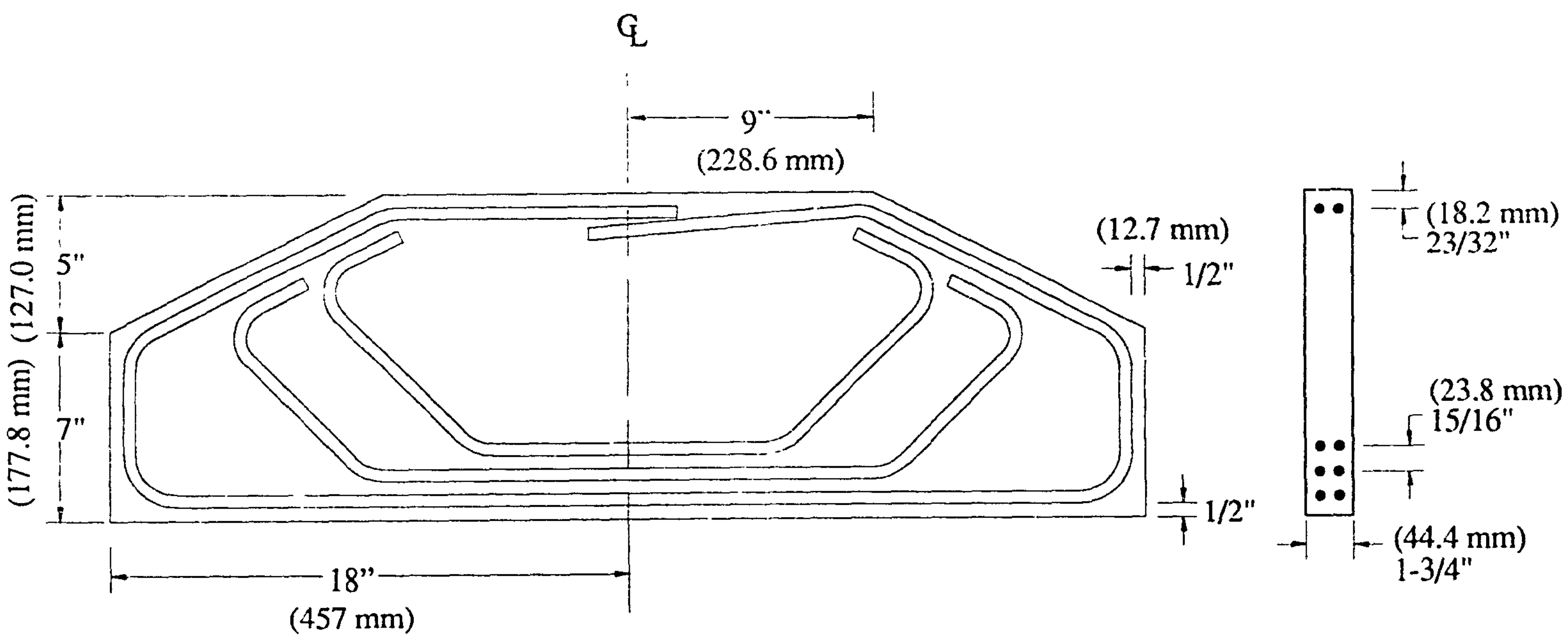


FIGURE 2.9 Reinforcement details for sample series 'P'

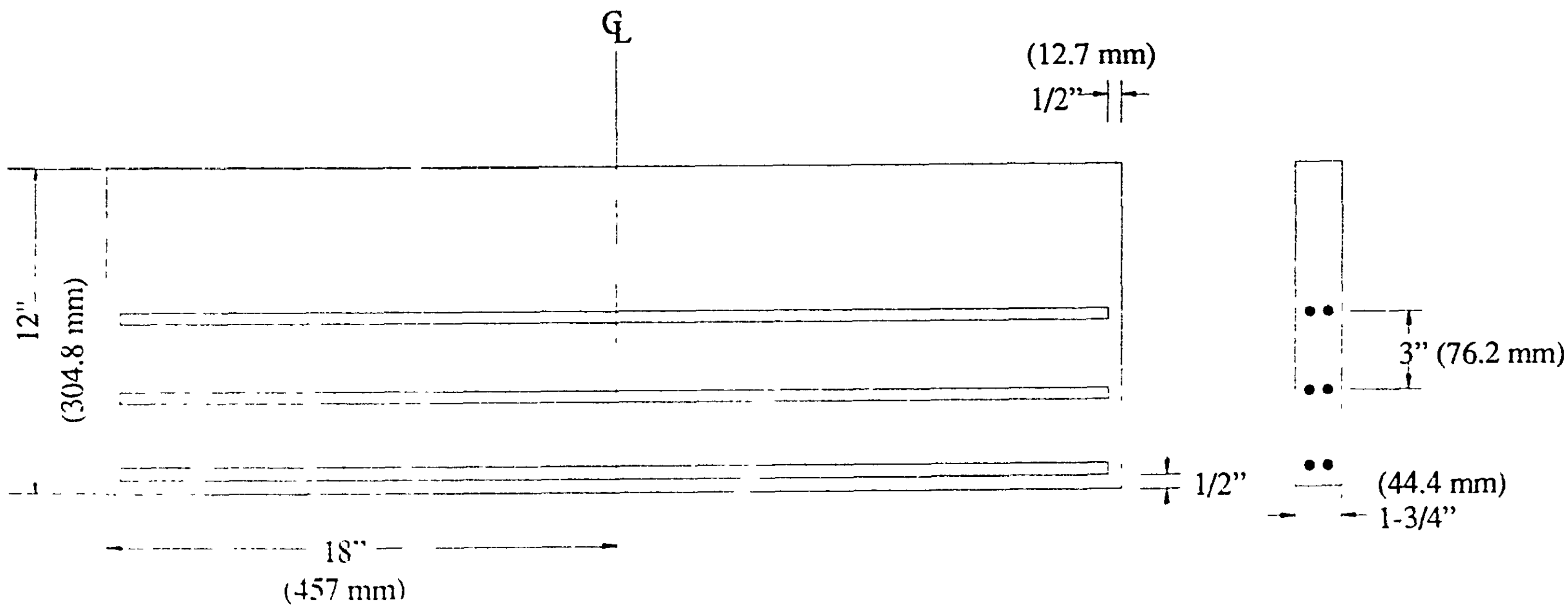


FIGURE 2.10 Reinforcement details for sample T1

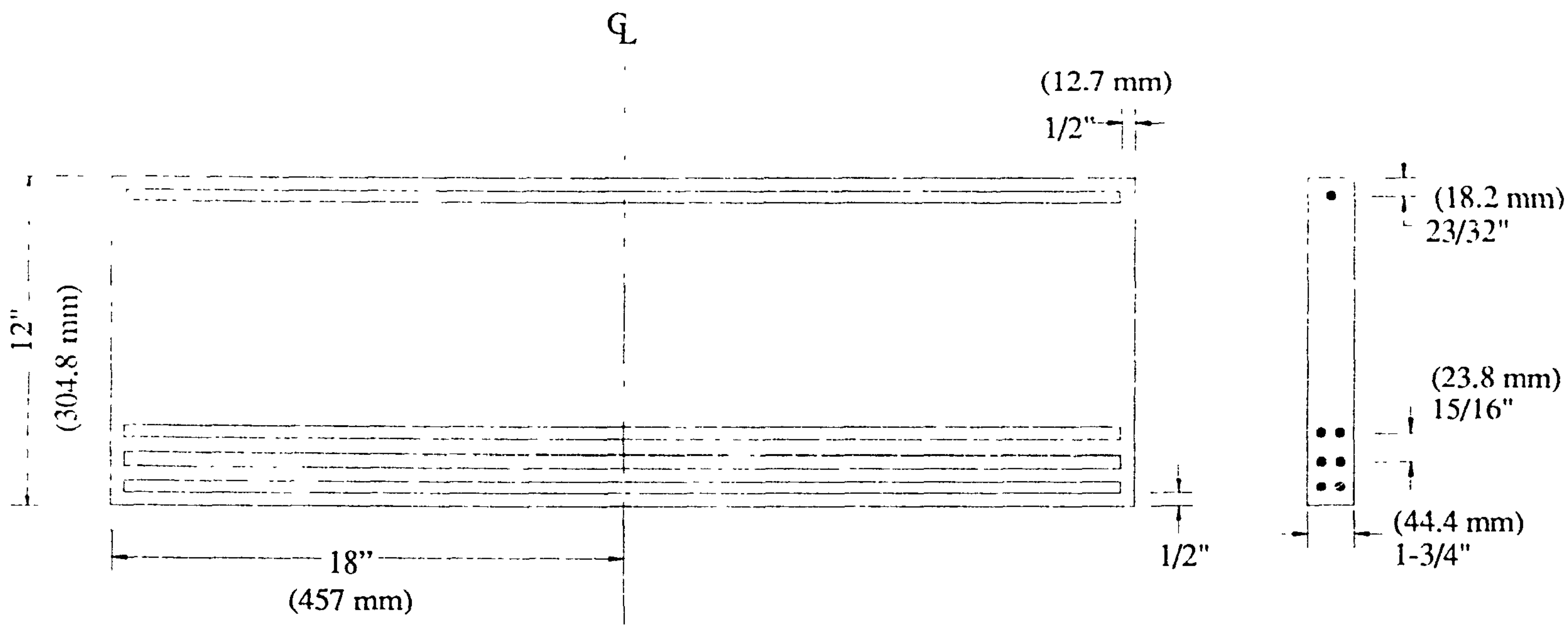
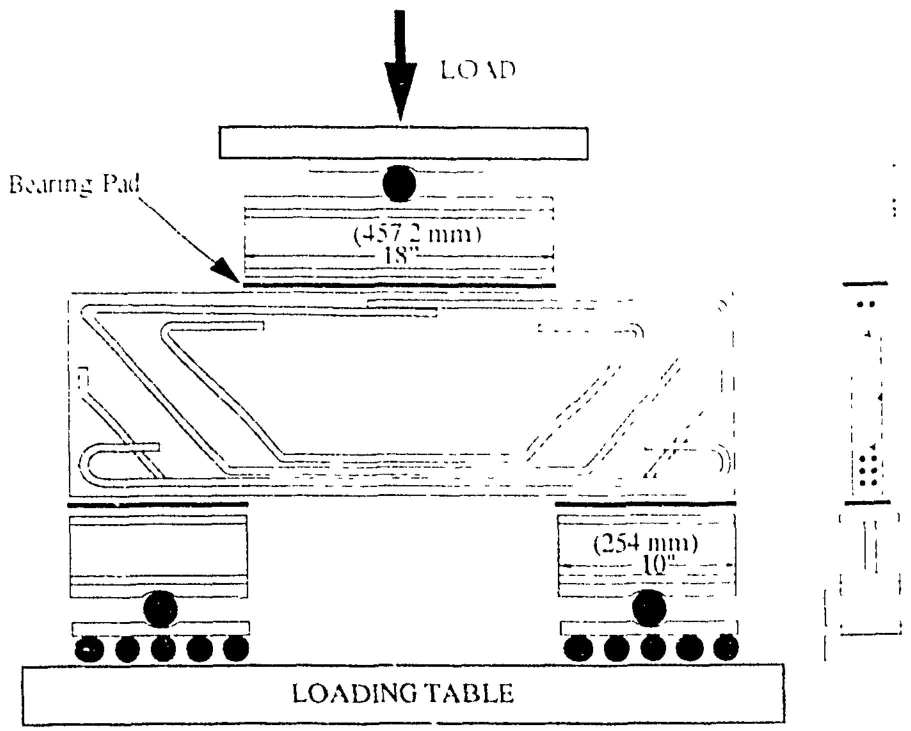


FIGURE 2.11 Reinforcement details for sample T2'



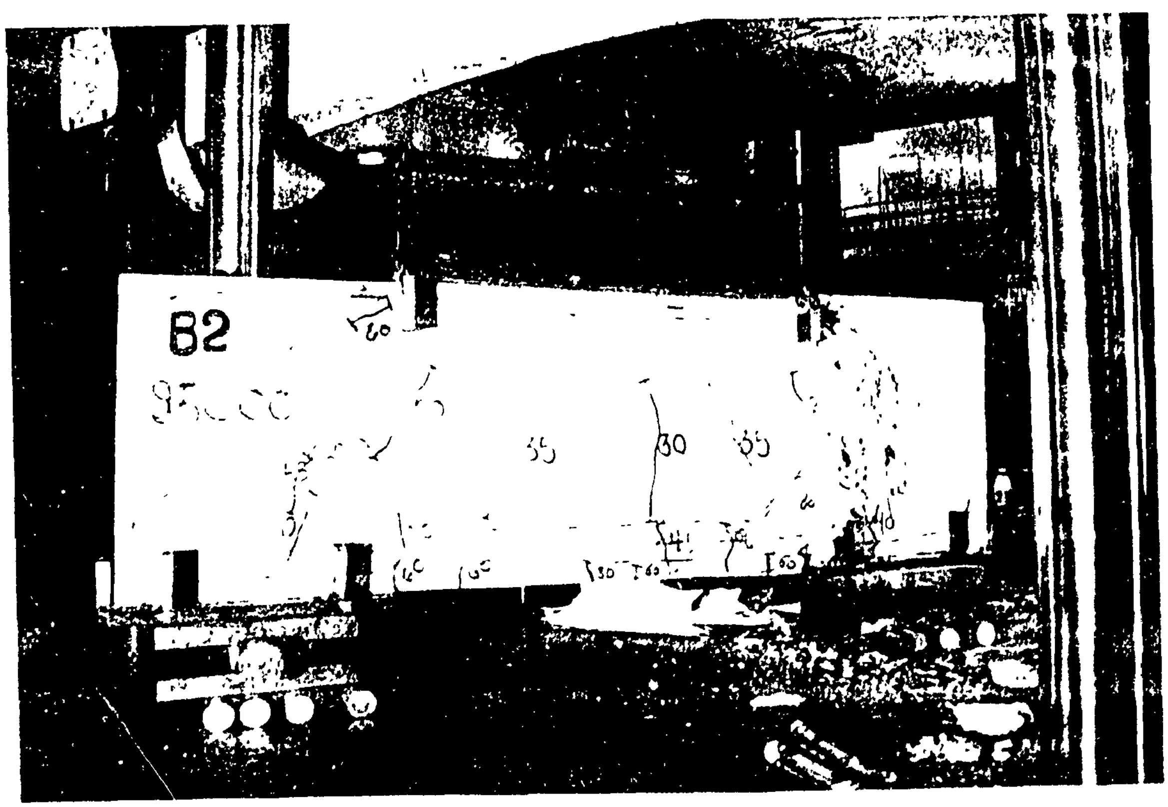
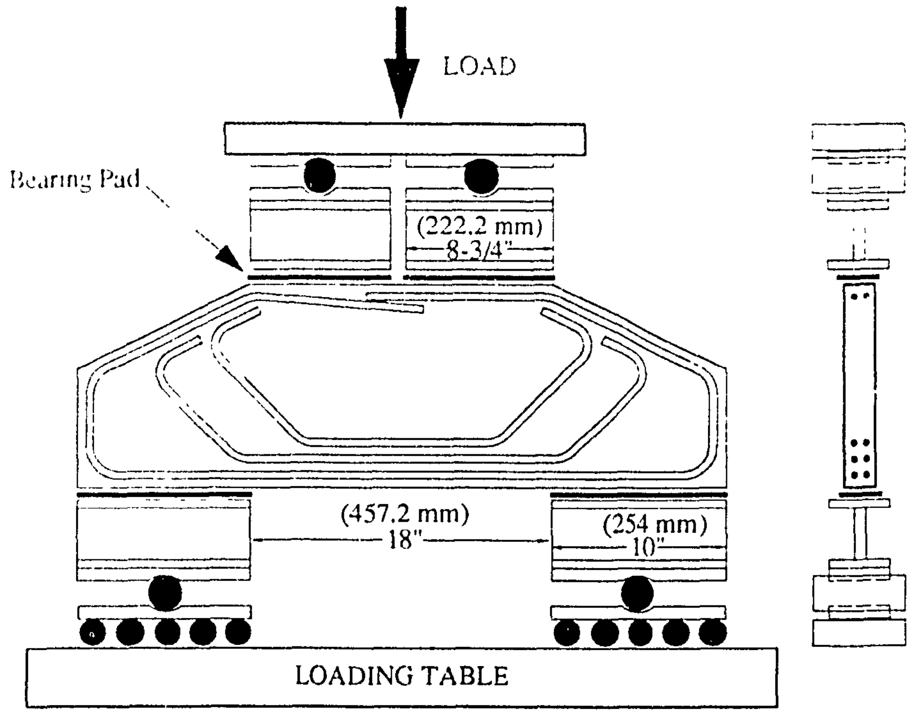


FIGURE 2.12 Details of loading scheme for Test Program I



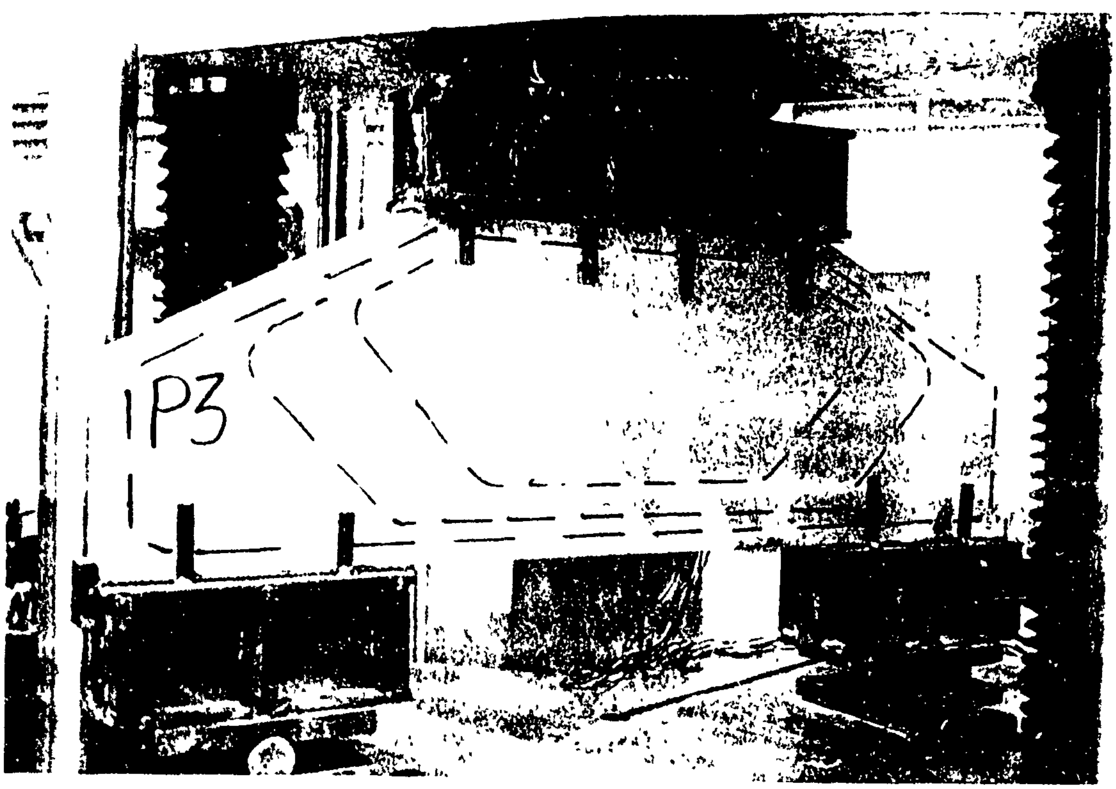


FIGURE 2.13 Details of loading scheme for Test Program II

CHAPTER III

PRE-CRACKING STRENGTH MODEL

3.1 General

Work models in the form of free body diagrams, (comprised of a concrete compression struts and steel tension ties) were used to analyze the test results. Two stages of loading were considered. The first stage examines the strength at inclined cracking of concrete. The second stage examines the ultimate capacity (after cracking). It should be noted here that the free body diagrams presented can be compared with multiple truss systems dating as far back as 1929 by Morsh[1] and by others [4],[17] and lately by [12].

Although the strength work models are being presented as work models for corbels, they can be applied to any reinforced concrete element subjected predominantly to shear. Therefore the use of the word "corbel" in this document encompasses all elements such as corbels or brackets, dapped ends, and deep beams. Only inclined cracking due to conventionally called "shear" is studied in this research.

3.2 Strength of Concrete at Cracking

Let us consider a non reinforced concrete corbel under pure shear action ($a<h$) (Fig. 3.1). The interface (line A-B) will be subjected to a biaxial stress state. The principal stresses will be acting under $\alpha=45^0$ and are comprised of orthogonal compression (σ_c) and tension (σ_t) stresses. Under pure shear these stresses are equal (Fig. 3.2).

If the strength limits for concrete subjected to biaxial stress is described by a straight line function as proposed by Zielinski[8],[9],[10] and Tasuji[18] (Fig 3.3), then the concrete corbel will fail by inclined cracking when the shearing stress reaches the capacity of concrete under tension-compression

$$v_{su} = f_{tc} \cong 0.9 f_{to} \quad [1]$$

where f_{to} is the uniaxial tensile strength limit of concrete.

If it is assumed that

$$f_{to} = 6\sqrt{f'_c} \quad [2]$$

$$f_{to} = 0.56\sqrt{f'_c} \qquad \text{(SI)} \quad [3]$$

then the inclined cracking and ultimate shear strength for the case of a non reinforced concrete corbel in terms of stress will be equal to:

$$v_{su} = 0.9 \times 6\sqrt{f'_c} = 5.4\sqrt{f'_c} \quad [4]$$

$$v_{su} = 0.9 \times 0.56\sqrt{f'_c} = 0.5\sqrt{f'_c} \qquad \text{(SI)} \quad [5]$$

The ultimate shear capacity can also be expressed as a force according to the free body diagram (Fig. 3.1) as follows:

$$V_u = \frac{C + T}{\sqrt{2}} \quad [6]$$

Given that under pure shear action compression equals tension (C=T) then

$$V_u = C\sqrt{2} = T\sqrt{2} \quad [7]$$

or in terms of stress

$$V_u = \sigma_c bh = \sigma_t bh \quad [8]$$

A non reinforced corbel will crack and fail when the shear force reaches its limit of:

$$V_u = v_{su}bh = 5.4\sqrt{f'_c}bh \qquad [9]$$

$$V_u = v_{su}bh = 0.5\sqrt{f'_c}bh \quad (SI) \qquad [10]$$

When reinforcement is present and sufficient to take the tension force T, cracking will not lead to the failure of the corbel. Only the reinforcement that intersects the cracks is useful. Reinforcement positioned perpendicular to the cracking will be the most effective. In general, the projection of the reinforcement to a line perpendicular to the crack should be considered (Fig. 3.4):

$$A_{wf} = A_{vf}\sin\alpha + A_{hf}\sin\alpha \qquad [11]$$

where α= the angle between the reinforcement bar and the cracks.

For horizontal (A_{hf}) and vertical (A_{vf}) reinforcement both at a 45^0 angle with the line perpendicular to the cracks, the projected area will be:

$$A_{wf} = \frac{A_{vf} + A_{hf}}{\sqrt{2}} \qquad [12]$$

$$\rho_{wf} = \frac{A_{wf}}{b\frac{h}{\sqrt{2}}} = \frac{\sqrt{2}A_{wf}}{bh} \qquad [13]$$

According to test results of Tasuji et al.[18], reinforced concrete under equal biaxial compression and tension stresses cracks when the strain in the tensile direction reaches a value of ε_{ct}=0.0001. Considering this strain value, at the moment of cracking the steel will be subjected to a stress of:

$$f_s = \varepsilon_t E_s = 0.0001 \times 29 \times 10^6 = 2900\text{psi} \qquad [14]$$

$$f_s = \varepsilon_t E_s = 0.0001 \times 200\text{GPa} = 20\text{MPa} \qquad [15]$$

The shear force at which a reinforced concrete corbel is expected to crack can be expressed in terms of concrete and steel stresses at the moment of cracking:

$$V_{ucr} = v_{su} bh + A_{wf} f_s = v_{su} bh + \frac{\rho_{wf} bh}{\sqrt{2}} f_s \qquad [16]$$

$$V_{ucr} = \left(v_{su} + \frac{\rho_{wf}}{\sqrt{2}} f_s \right) bh \qquad [17]$$

$$V_{ucr} = (5.4\sqrt{f'_c} + 2050\rho_{wf})\, bh \qquad [18]$$

$$V_{ucr} = (0.5\sqrt{f'_c} + 14\rho_{wf})\, bh \qquad (\text{SI}) \qquad [19]$$

For the case when only horizontal reinforcement is present (eq. [12]):

$$A_{wf} = \frac{A_{hf}}{\sqrt{2}} \qquad [20]$$

combining equation [20] and [13]

$$\rho_{wf} = \frac{A_{hf}}{\sqrt{2}} \bullet \frac{\sqrt{2}}{bh} = \rho_{hf} \qquad [21]$$

$$V_{ucr} = (5.4\sqrt{f'_c} + 2050\rho_{hf})\, bh \qquad [22]$$

$$V_{ucr} = (0.5\sqrt{f'_c} + 14\rho_{hf})\, bh \qquad (\text{SI}) \qquad [23]$$

3.3 Minimum Required Reinforcement to Avoid Sudden Failure

Having defined the cracking strength of a reinforced concrete corbel it is important to establish the minimum reinforcement needed to ensure that the structure does not collapse at the instant of crack appearance. As indicated in Figure 3.1, the total tension force needed to be carried by the steel immediately after cracking to avoid sudden failure is:

$$T = \frac{V_{ucr}}{\sqrt{2}} = (5.4\sqrt{f'_c} + 2050\rho_{wf})\frac{bh}{\sqrt{2}} \qquad [24]$$

$$T = \frac{V_{ucr}}{\sqrt{2}} = (0.5\sqrt{f'_c} + 2050\rho_{hf})\frac{bh}{\sqrt{2}} \qquad (SI) \qquad [25]$$

In order to avoid failure the tensile capacity provided by the steel should be greater than this force.

$$T_y = A_{wf}f_y = \rho_{wf}\frac{bh}{\sqrt{2}}f_y \qquad [26]$$

comparing equations [24] and [26]

$$\frac{bh}{\sqrt{2}}(5.4\sqrt{f'_c} + 2050\rho_{wf}) \leq \rho_{wf}\frac{bh}{\sqrt{2}}f_y \qquad [27]$$

$$\rho_{wf}f_y - 2050\rho_{wf} \geq 5.4\sqrt{f'_c} \qquad [28]$$

From the above equations, we can establish the minimum required reinforcement as:

$$\rho_{wf} \geq \frac{5.4\sqrt{f'_c}}{f_y - 2050} \quad \text{or} \quad A_{wf} \geq \frac{5.4\sqrt{f'_c}\,bh}{\sqrt{2}\,(f_y - 2050)} \qquad [29]$$

$$\rho_{wf} \geq \frac{0.5\sqrt{f'_c}}{f_y - 14} \quad \text{or} \quad A_{wf} \geq \frac{0.5\sqrt{f'_c}\,bh}{\sqrt{2}\,(f_y - 14)} \qquad (SI) \qquad [30]$$

Where A_{wf} and ρ_{wf} describe the area and ratio of the 45^0 inclined reinforcement respectively.

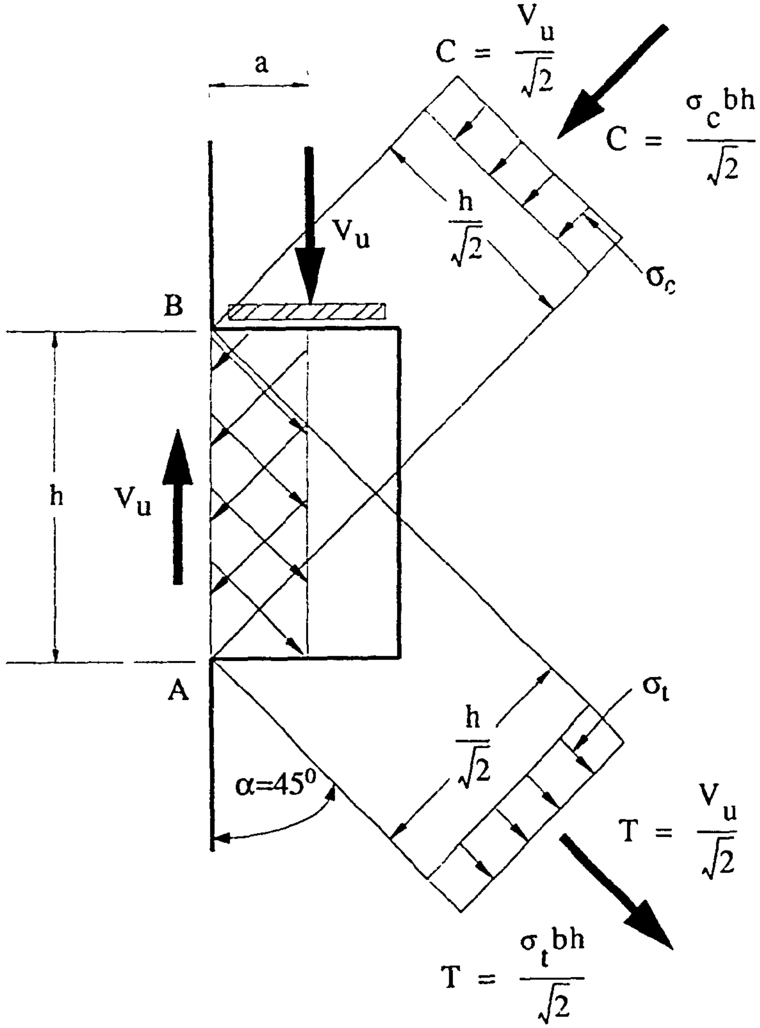
For the case of only horizontal reinforcement, the minimum amount of reinforcement to avoid sudden failure is described by equations [29] and [30] with A_{wf} and ρ_{wf} replaced by $A_{hf}/\sqrt{2}$ and ρ_{hf} respectively.

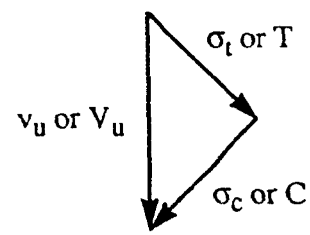
$$\rho_{hf} \geq \frac{5.4\sqrt{f'_c}}{f_y - 2050} \quad \text{or} \quad A_{hf} \geq \frac{5.4\sqrt{f'_c}\,bh}{f_y - 2050} \qquad [31]$$

$$\rho_{hf} \geq \frac{0.5\sqrt{f'_c}}{f_y - 14} \quad \text{or} \quad A_{hf} \geq \frac{0.5\sqrt{f'_c}\,bh}{f_y - 14} \quad \text{(SI)} \qquad [32]$$

3.4 Applying the Model to the Test Results

The cracking strength of all test samples was calculated using equations [29] and [31]. For the test series with inclined reinforcement it was necessary to combine these two equations. This was done because these samples had both inclined and horizontal reinforcement. In the case of the test samples with horizontal with vertical stirrups, the area of steel was determined by applying equation [12]. The calculated cracking strength and the experimental data is shown in Tables 3.1 to 3.3. The ratio of experimental data to the theoretical calculations is also provided. From the average values of these ratios, it can seen that the proposed model for the cracking strength of reinforced concrete corbels can predict the cracking load with a high degree of accuracy. It is also interesting to note that the average ratio of the nominal shear cracking stress to the compressive strength of concrete (v_{lcr}/f'_c) for the samples with horizontal and vertical reinforcement is the same as the samples with only horizontal reinforcement. This implies that vertical stirrups do not contribute to the cracking strength of reinforced concrete corbels. The data also shows that the average ratio of v_{icr}/f'_c for each of the three test groups varies from 0.18 to 0.25. If the effective depth, instead of the full beam depth was considered, the results would yield higher ratios.





$$\sigma_c = -\sigma_t$$

$$T = -C$$

$$v_u = \frac{V_u}{bh}$$

$$\frac{a}{h} \leq 1$$

$$V_u = \frac{C + T}{\sqrt{2}} = \sqrt{2}C = \sqrt{2}T$$

FIGURE 3.1 Work model for a non reinforced concrete corbel under "pure shear"

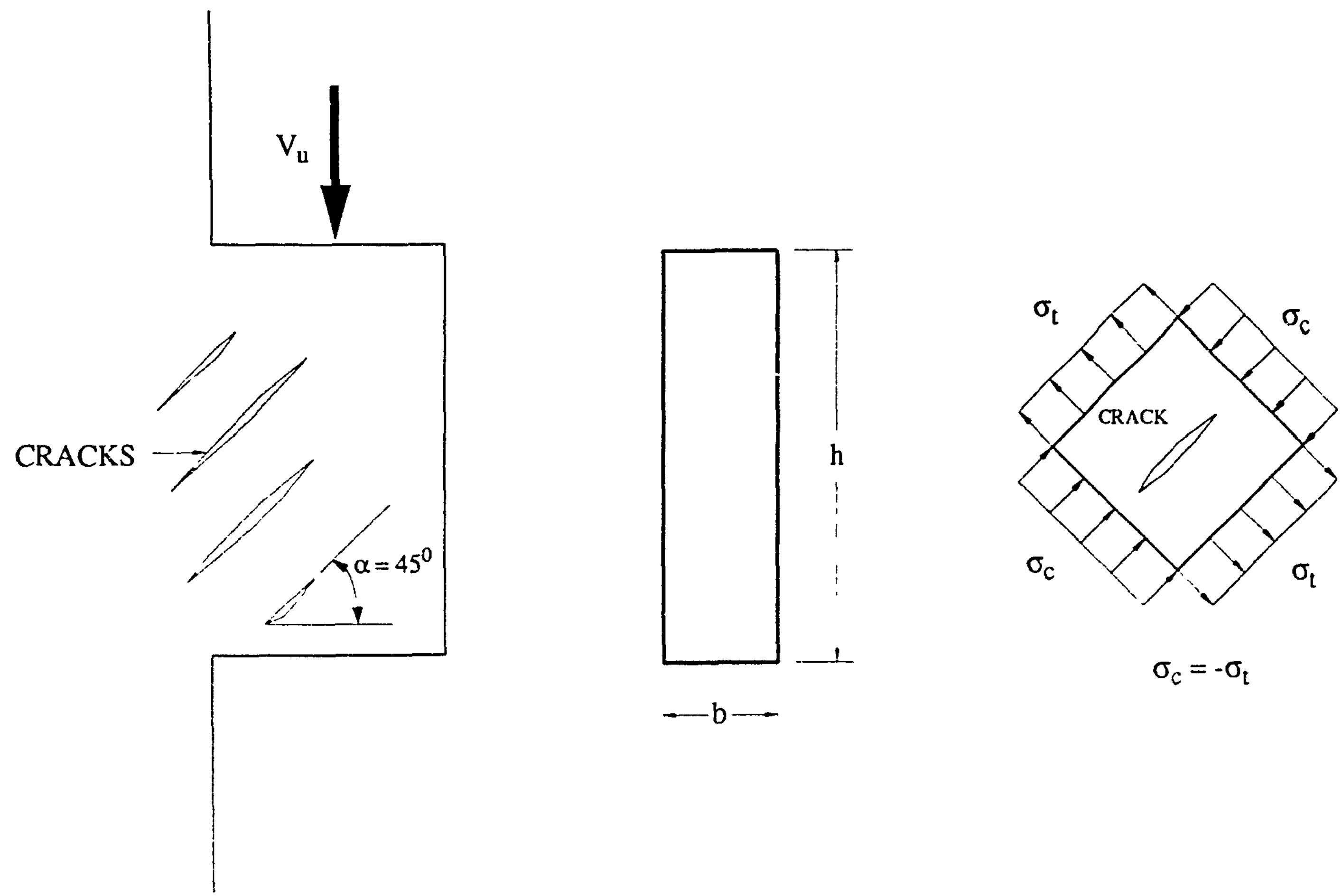


FIGURE 3.2 Principal biaxial stress state of a concrete corbel subjected to shear load

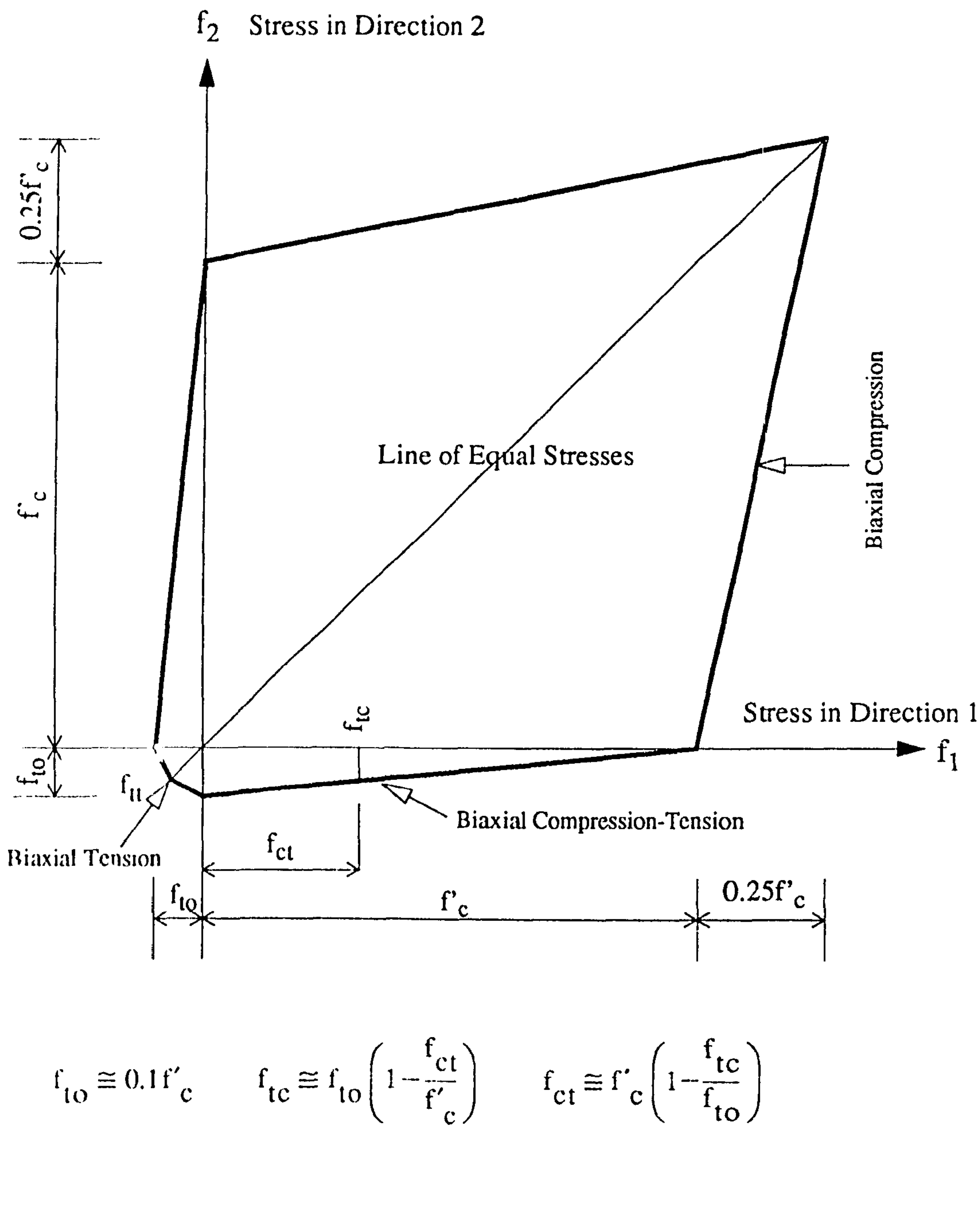
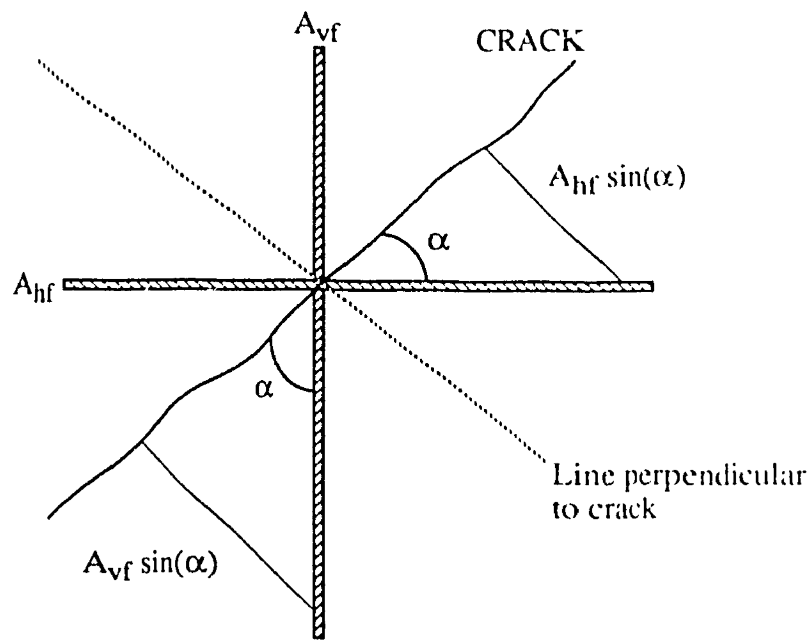


FIGURE 3.3 Biaxial strength envelope for concrete



$$A_{wf}=A_{hf}\sin\alpha + A_{vf}\sin\alpha$$

FIGURE 3.4 Projection of reinforcement to a line perpendicular to cracks

TABLE 3.1 Cracking data for horizontally reinforced concrete corbels

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_{lcr} kips (kN)	V_{cr} based on calculated capacity kips (kN)	V_{lcr}/V_{cr}	$v_{lcr}=V_{lcr}/bh$ psi (MPa)	v_{lcr}/f'_c
1	2	3	4	5	6	7=5/6	8=6/4	9=8/2
S1	3791.4 (26.1)	69000 (475.7)	21.4 (138)	10.0 (44.4)	9.7 (43.1)	1.03	467.3 (3.22)	0.12
S3	3791.4 (26.1)	69000 (475.7)	21.4 (138)	10.0 (44.4)	9.7 (43.1)	1.03	467.3 (3.22)	0.12
S4	4566.0 (31.5)	40000 (275.8)	21.3 (137)	12.5 (55.6)	10.3 (45.8)	1.21	586.8 (4.05)	0.13
S5	4566.0 (31.5)	40000 (275.8)	21.4 (138)	12.5 (55.6)	10.4 (46.2)	1.20	584.1 (4.03)	0.13
H1	3346.3 (23.1)	40000 (275.8)	21.8 (140)	15.0 (66.7)	8.8 (39.1)	1.70	688.1 (4.74)	0.20
H2	3346.3 (23.1)	40000 (275.8)	21.8 (140)	17.5 (77.8)	8.8 (39.1)	1.99	802.7 (5.53)	0.24
H3	3346.3 (23.1)	40000 (275.8)	21.8 (140)	13.5 (60.0)	8.8 (39.1)	1.53	619.3 (4.27)	0.18
R1	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	15.0 (66.7)	7.7 (34.2)	1.95	714.3 (4.92)	0.27
R2	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	15.0 (66.7)	7.7 (34.2)	1.95	714.3 (4.92)	0.27
R3	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	17.5 (77.8)	7.7 (34.2)	2.27	833.3 (5.74)	0.32
Q1	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	12.5 (55.6)	8.4 (37.4)	1.49	595.2 (4.10)	0.18
Q2	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	12.5 (55.6)	8.4 (37.4)	1.49	595.2 (4.10)	0.18
Q3	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	10.0 (44.4)	8.4 (37.4)	1.19	476.2 (3.28)	0.15
T1	1176.0 (8.1)	60000 (414.0)	21.0 (135.5)	12.5 (55.6)	5.8 (25.8)	2.1	595.2 (4.1)	0.50
T2	1176.0 (8.1)	60000 (414.0)	21.0 (135.5)	12.5 (55.6)	5.8 (25.8)	1.9	535.7 (3.7)	0.45
average values						1.60	618.3 (4.26)	0.23

TABLE 3.2 Cracking data for horizontal and vertically reinforced concrete corbels

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_{lcr} kips (kN)	V_{cr} based on calculated capacity kips (kN)	V_{lcr}/V_{cr}	$v_{lcr}=V_{lcr}$ /bh psi (MPa)	v_{lcr}/f'_c
1	2	3	4	5	6	7=5/6	8=6/4	9=8/2
C1	3346.3 (23.1)	69000 (475.7)	21.9 (132.5)	12.5 (55.6)	11.9 (53.1)	1.05	570.8 (3.9)	0.17
C2	3346.3 (23.1)	69000 (475.7)	21.6 (140)	12.5 (55.6)	11.8 (52.6)	1.06	578.7 (4.0)	0.17
C3	3346.3 (23.1)	69000 (475.7)	21.6 (140)	12.5 (55.6)	11.8 (52.6)	1.06	578.7 (4.0)	0.17
V1	3607.7 (24.9)	40000 (275.8)	21.7 (140)	10.0 (44.4)	10.5 (46.9)	0.95	460.8 (3.2)	0.13
V2	3607.7 (24.9)	40000 (275.8)	21.7 (140)	15.0 (66.7)	10.5 (46.9)	1.43	691.2 (4.8)	0.19
V3	3607.7 (24.9)	40000 (275.8)	21.7 (140)	20.0 (88.9)	10.5 (46.9)	1.90	921.6 (6.4)	0.25
average values						1.24	633.6 (4.36)	0.18

TABLE 3.3 Cracking data for inclined reinforced concrete corbels

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_{icr} kips (kN)	V_{cr} based on calculated capacity kips (kN)	V_{icr}/V_{cr}	$v_{icr}=V_{icr}/bh$ psi (MPa)	v_{icr}/f'_c
1	2	3	4	5	6	7=5/6	8=6/4	9=8/2
B2	3752.5 (25.9)	73000 (503.3)	21.6 (140)	17.5 (77.8)	9.6 (42.9)	1.82	810.2 (5.6)	0.22
B4	4329.0 (29.8)	73000 (503.3)	21.4 (138)	20.0 (88.9)	10.1 (44.9)	1.98	934.6 (6.4)	0.22
B5	4329.0 (29.8)	73000 (503.3)	21.3 (137)	10.0 (44.4)	10.0 (44.5)	1.00	469.5 (3.2)	0.11
B6	2613.0 (18.0)	40000 (275.8)	22.5 (145)	20.0 (88.9)	8.8 (39.1)	2.27	952.4 (6.6)	0.36
B7	2150.0 (14.8)	40000 (275.8)	21.0 (135)	15.0 (66.7)	8.2 (36.5)	1.83	714.3 (4.9)	0.33
B8	2150.0 (14.8)	40000 (275.8)	21.0 (135)	12.5 (55.6)	8.2 (36.5)	1.52	595.2 (4.1)	0.28
$A2_{78}$	2150.0 (14.8)	60000 (414.0)	21.0 (136)	32.5 (144.5)	8.3 (36.9)	3.91	1547.6 (10.7)	0.72
$A3_{78}$	2150.0 (14.8)	60000 (414.0)	21.0 (136)	29.0 (129.0)	7.4 (32.9)	3.92	1381.0 (9.5)	0.64
$B4_{78}$	4846.0 (33.4)	60000 (414.0)	21.0 (136)	25.0 (111.2)	11.0 (48.9)	2.27	1190.5 (8.2)	0.24
$B5_{78}$	4846.0 (33.4)	60000 (414.0)	21.0 (136)	25.0 (111.2)	11.0 (48.9)	2.27	1190.5 (8.2)	0.24
$B6_{78}$	4846.0 (33.4)	60000 (414.0)	21.0 (136)	12.5 (55.6)	10.0 (44.5)	1.25	595.2 (4.1)	0.12
P1	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	15.0 (66.7)	8.5 (37.8)	1.76	714.3 (4.9)	0.25
P2	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	15.0 (66.7)	8.5 (37.8)	1.76	714.3 (4.9)	0.25
P3	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	15.0 (66.7)	8.5 (37.8)	1.76	714.3 (4.9)	0.25
average values						2.09	714.3 (4.9)	0.25

CHAPTER IV

ULTIMATE STRENGTH (POST CRACKING)

4.1 Ultimate Strength (After Cracking)

After inclined cracking (as described earlier) the system can continue to function if proper and sufficient reinforcement is present. The corresponding multiple free body diagrams for reinforced concrete corbels consist of inclined compression struts and horizontal or inclined steel tension ties (Figs. 4.1, 4.2 respectively).

The free body diagrams presented here require the definition of the ultimate compression strength of the concrete struts, C, and the ultimate tensile strength of the steel ties, T. Because of the presence of cracks it will be assumed that the concrete compressive strength will be reduced to f_{cs}=0.85f'_c. This assumption is based on the notion that between the inclined cracks the concrete will act as columns with the reinforcement providing the required stability. The total capacity of the compression struts can be defined by the free body diagram as:

$$C = f_{cs}\frac{bh}{\sqrt{2}} = 0.85f'_c\frac{bh}{\sqrt{2}} = 0.6f'_c bh \qquad [33]$$

The capacity of the steel tension tie is defined as the area of steel times the yield capacity.

$$\text{for horizontal reinforcement} \quad T = A_{hf}f_y$$
$$\text{for inclined reinforcement} \quad T = A_{wf}f_y \qquad [34]$$

To eliminate extensive deformations and weakening of the free body work models, it is proposed to limit the maximum usable strength of steel to f_y=40000 psi (f_y=27.6

MPa), which corresponds to a maximum strain of ε_{ctu}=0.00138.

4.2 Ultimate Capacity for the Case of Only Horizontal Reinforcement

From the free body diagram (Fig. 4.1), force equilibrium for a corbel with horizontal reinforcement is satisfied when:

$$V_u = T = \frac{C}{\sqrt{2}} \quad [35]$$

substituting with the defined capacities of the compression strut and tension tie the shear capacity can be defined by the governing capacity:

$$V_u = T = A_{hf} f_y$$

or

$$V_u = \frac{C}{\sqrt{2}} = \frac{0.6f'_c bh}{\sqrt{2}} = 0.424f'_c bh \quad [36]$$

in terms of compressive stress in concrete

$$v_u = \frac{V_u}{bh} = 0.424f'_c \quad [37]$$

in terms of ultimate (yield) tensile stress in steel

$$v_u = \frac{V_u}{bh} = \frac{A_{hf} f_y}{bh} = \rho_{hf} f_y \quad [38]$$

4.2.1 The Required Horizontal Reinforcement

For design practice it is important to be able to calculate the required reinforcement. The above equations show that the capacity of a corbel is governed by the strength either it's compression strut or tension tie. Therefore to determine the required horizontal reinforcement it is important that the capacity of the compression strut is also considered. From equation [36], the required area of steel can be determined as:

$$A_{hf} = \frac{V_u}{f_y} \qquad [39]$$

but, the shear load cannot exceed the capacity of the concrete strut (eq. [36]):

$$V_u \leq 0.424 f'_c bh \qquad [40]$$

Also, the required reinforcement cannot be less than the minimum amount as defined earlier by equations [31] and [32] or the corbel will fail suddenly at crack appearance.

4.2.2 Application of Model to Test Results

Since the behavior of the samples with vertical stirrups was very similar to the samples with only horizontal reinforcement, it was concluded that vertical stirrups do not contribute to the overall strength of reinforced concrete corbels. The model for horizontal reinforcement neglecting the vertical reinforcement was applied to test series 'C' and 'V' as well as the series 'S', 'H', 'R', 'Q' and 'T'. A sample calculation is provided below.

The average failure of series 'S' was 38.7 kips (172 kN). Ultimate shear capacity in terms of concrete or steel capacity can be determined from equation [36]. The average value of concrete strength f'_c=4178.7 psi (28.8 MPa) and shear area bh=21.4 in^2 (137.8 cm^2) were applied to the test samples of series 'S'. The capacity of the corbel in terms of concrete strength can be determined as:

$$V_{uc} = 0.424 f'_c bh = 0.424 \times 4178.7 \times 21.4 = 37.5\text{kips} \qquad (166.6\text{kN})$$

in terms of steel capacity (using average values) the capacity of the same corbel is:

$$V_{us} = A_{hf} f_y = 0.933 \times 54500 = 50.8\text{kips} \qquad (226.0\text{kN})$$

or based on the proposed steel capacity limit of f_y=40000 psi (f_y=27.6 MPa).

$$V_{up} = A_{hf}f_y = 0.933 \times 40000 = 37.8\text{kips} \qquad (166.0\text{kN})$$

From these calculations it can be recorded that series 'S' failed when reaching the concrete capacity. The calculated capacity of 37.5 kips (166.6 kN) is very close to the actual failure of 38.7 kips (172 kN). It should be noted that because of the shallow concrete cover, the bottom row (top for beam) of reinforcement was excluded from the calculations since the small amount of concrete would not allow the development of the 45^0 inclined compression force and hence not contribute to the overall strength. The above calculated strength results, together with the calculated capacities of test series 'H', 'R', 'Q', 'T', 'C', and 'V' are shown in Tables 4.1 to 4.6.

Applying the shear strength limits as prescribed by the ACI 318-83 code to the same samples, calculated strengths are:

$$V_n \le 0.2f'_c b_w d \qquad \text{or} \qquad \le 800 b_w d$$

$$V_n \le 0.2f'_c b_w d \le 0.2 \times 4178.7 \times 21.4 = 17.9\text{kips} \qquad (79.5\text{kN})$$

$$\text{or} \qquad V_n \le 800 b_w d \le 800 \times 21.4 = 17.1\text{kips} \qquad (76.1\text{kN})$$

The calculated shear capacity for series 'S' as described by the American code is 17.1 kips (76.1 kN). The actual shear capacity of the test samples was more than twice the allowed by ACI 318-83. This demonstrates the need for developing new techniques in determining shear capacity of corbels. Not only will this lead to more economic designs, but it will also lead to a better understanding of the role played by concrete and steel under shear.

The ratio of theoretical to experimental shear capacities has been provided on Tables 4.1 to 4.7. It is clear from these ratios that the proposed model for horizontally reinforced concrete corbels can predict the ultimate shear capacity of such elements with a high degree of accuracy.

Test results for series 'H' and 'V' indicated shear capacities much greater than those calculated based on either steel and concrete capacities. The model for uniformly distributed horizontal reinforcement may be applied conservatively in the case of grouped reinforcement.

4.3 Ultimate Capacity for the Case of Inclined Reinforcement

From the free body diagram (Fig. 4.2) force equilibrium can be reached when:

$$V_u = \frac{(C+T)}{\sqrt{2}} \qquad [41]$$

as defined earlier $C=0.6f'_c bh$ and $T=A_{wf}f_y$

$$V_u = \frac{(0.6f'_c bh + A_{wf}f_y)}{\sqrt{2}} = 0.424f'_c bh + \frac{A_{wf}f_y}{\sqrt{2}} \qquad [42]$$

considering shear stress

$$v_u = \frac{V_u}{bh} = 0.424f'_c + \frac{A_{wf}f_y}{\sqrt{2}bh} \qquad [43]$$

$$v_u = 0.424f'_c + 0.5\rho_{wf}f_y \qquad [44]$$

At the balanced condition the strength of the compression struts and tensile ties are equal. It is therefore possible to write the equations for maximum shear capacity based on either tension or compression capacity as follows.

4.3.1 In Terms of Tension (Steel) Capacity

$$V_u = \sqrt{2}T = \sqrt{2}A_{wf}f_y = \rho_{wf}f_y bh \qquad [45]$$

$$v_u = \frac{V_u}{bh} = \rho_{wf}f_y \qquad [46]$$

4.3.2 In Terms of Compression (Concrete) Capacity

$$V_u = \sqrt{2}C = \sqrt{2}(0.6f'_c bh) = 0.85f'_c bh \qquad [47]$$

in terms of stress:

$$v_u = \frac{V_u}{bh} = 0.85f'_c \qquad [48]$$

4.3.3 The Required Amount of Reinforcement

The required amount of reinforcement can be determined from equation [45] as:

$$A_{wf} = \frac{V_u}{\sqrt{2}f_y} \qquad [49]$$

but,

$$V_u = v_u bh \le 0.85f'_c bh \qquad [50]$$

and not less than the minimum as defined by equations [29] and [30].

Maximum shear capacity is based on the compression and tension stresses being equally carried by the concrete and steel respectively. It is then possible to determine the maximum usable amount of web reinforcement based on the compressive strength of concrete by comparing each component from equation [44].

$$0.424f'_c bh = \frac{A_{wf}f_y}{\sqrt{2}} \qquad [51]$$

The maximum usable amount of inclined reinforcement can be defined as:

$$A_{wf} \le 0.6\frac{f'_c}{f_y}bh \qquad [52]$$

$$\rho_{wf} \leq 0.85 \frac{f'_c}{f_y} \qquad [53]$$

For example, if $f'_c/f_y=0.1$ the maximum usable amount of web reinforcement would be $\rho_{wf}=0.085$. Increasing the amount of web reinforcement above this will not increase the shear strength of the corbel since the capacity will be limited by the concrete. It should be noted that the above established amount of reinforcement of $\rho_{wf}=0.085=8.5\%$ is very high and in practice difficult if not impossible to arrange.

4.3.4 Application of Model to Test Results

The average failure of samples B6, B7, B8 with 45^0 inclined reinforcement was determined to be 41.3 kips (184.0 kN). Using equation [47], the shear capacity in terms of concrete capacity can be determined as:

$$V_{uc} = 0.85 \times 2613 \times 21.5 = 47.7 \text{kips} \qquad (212\text{kN})$$

To calculate the steel capacity it is necessary to combine the equations [36], [45] for horizontal and inclined reinforced corbels as the samples tested had four (4) horizontal and two (2) inclined bars crossing the shear interface as follows:

$$V_{us} = A_{hf}f_y + \sqrt{2}A_{wf}f_y$$

Here, A_{wf}, is the area of two inclined bars, while A_{hf} is the area of four horizontal bars.

$$V_u = (4 \times 0.155 + 2 \times \sqrt{2} \times 0.155) \times 40000 = 42.4 \text{kips} \qquad (188.6\text{kN})$$

The calculations indicate that the failure of these samples was due to the steel reaching its capacity. As shown on Table 4.8, samples B4, B5 which did not fail, were loaded below their calculated capacities. Sample B2 failed before reaching its concrete or steel capacity, but it should be noted that it failed after reaching its steel capacity limit based on f_y=40000 psi (27.6 MPa). Since the failure of the samples in this series was primarily due to yielding of steel, values for 'nominal' shear stress (v_u) and ratios of v_u/f'_c are lower than had the concrete reached its ultimate capacity. All calculated values for the inclined series are shown on Table 4.8 to 4.10.

In order to test the accuracy of the models presented here, the ratio of actual failure load to the governing calculated capacity has been reported in Tables 4.1 to 4.10. As can be seen from these results, the models can be applied with a high degree of certainty. From the free body diagrams it was shown that inclined reinforcement is $\sqrt{2}$ times more economical than horizontal reinforcement. It should be emphasized that no capacity reduction factors were applied to the calculations presented. For design practice both the capacity reduction factors as well as load factors must be considered as required by the American or Canadian Codes.

Currently shear capacity for corbels and deep beams is limited to $V_n \leq 0.2f'_c b_w d$ or $v_u/f'_c \leq 0.2$ and $V_n \leq 800 b_w d$ by the American code (ACI 318-83 clause 11.9.3.2.1), and to $V_r \leq 0.25f'_c A_{cv}$ or $v_u/f'_c \leq 0.25$ by the Canadian code (CAN3-A23.3-M84 clause 11.7.5). These limits are even lower if referred to by h = full beam depth instead of d = effective depth. If it is assumed that for deep beams $d \approx 0.8 - 0.9h$ then $V_n \leq 0.16 - 0.18f'_c b_w h$ and $V_r \leq 0.2 - 0.225f'_c bh$. The findings suggests that the above limits could be significantly increased for corbels, deep beams and dapped ends.

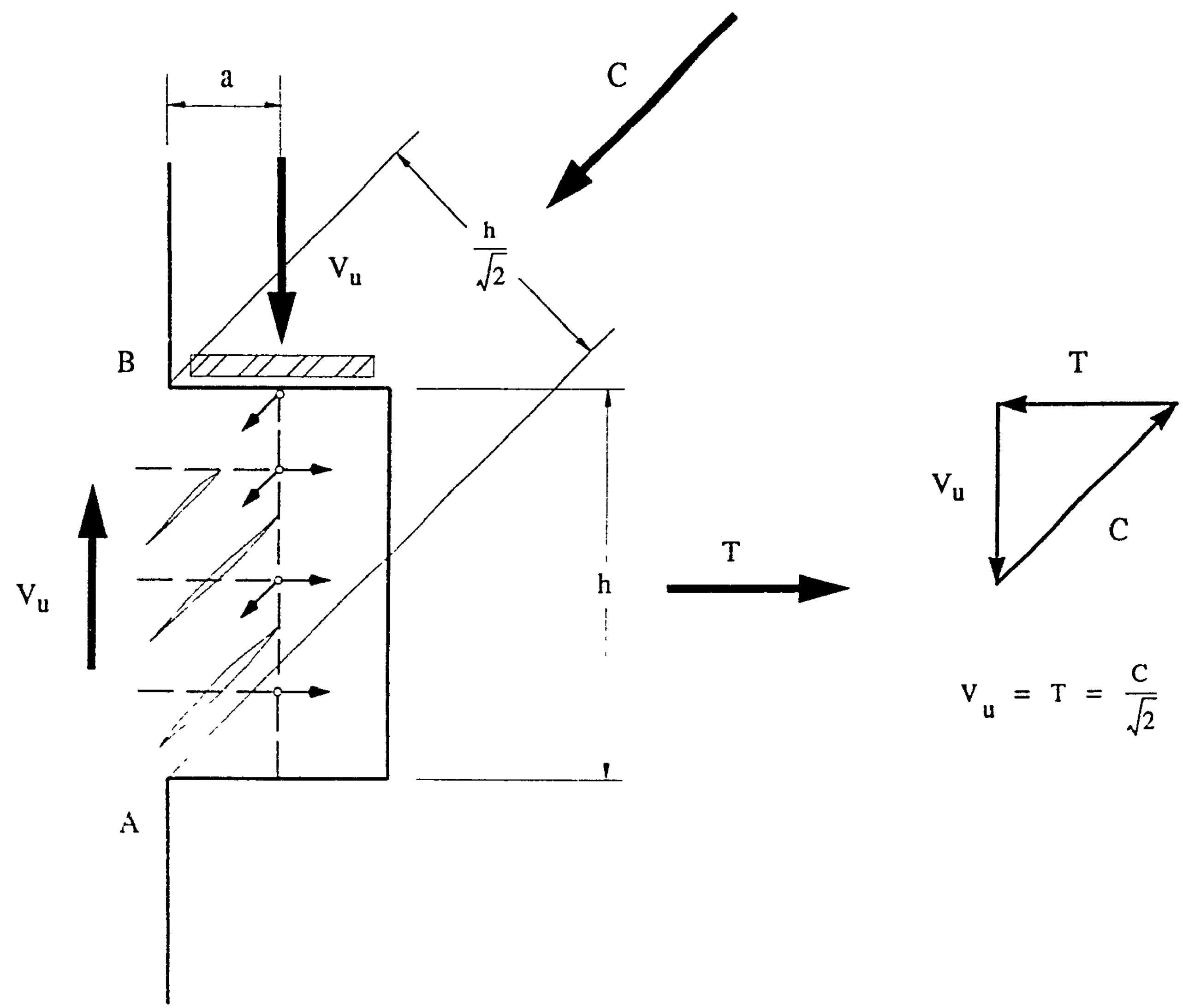
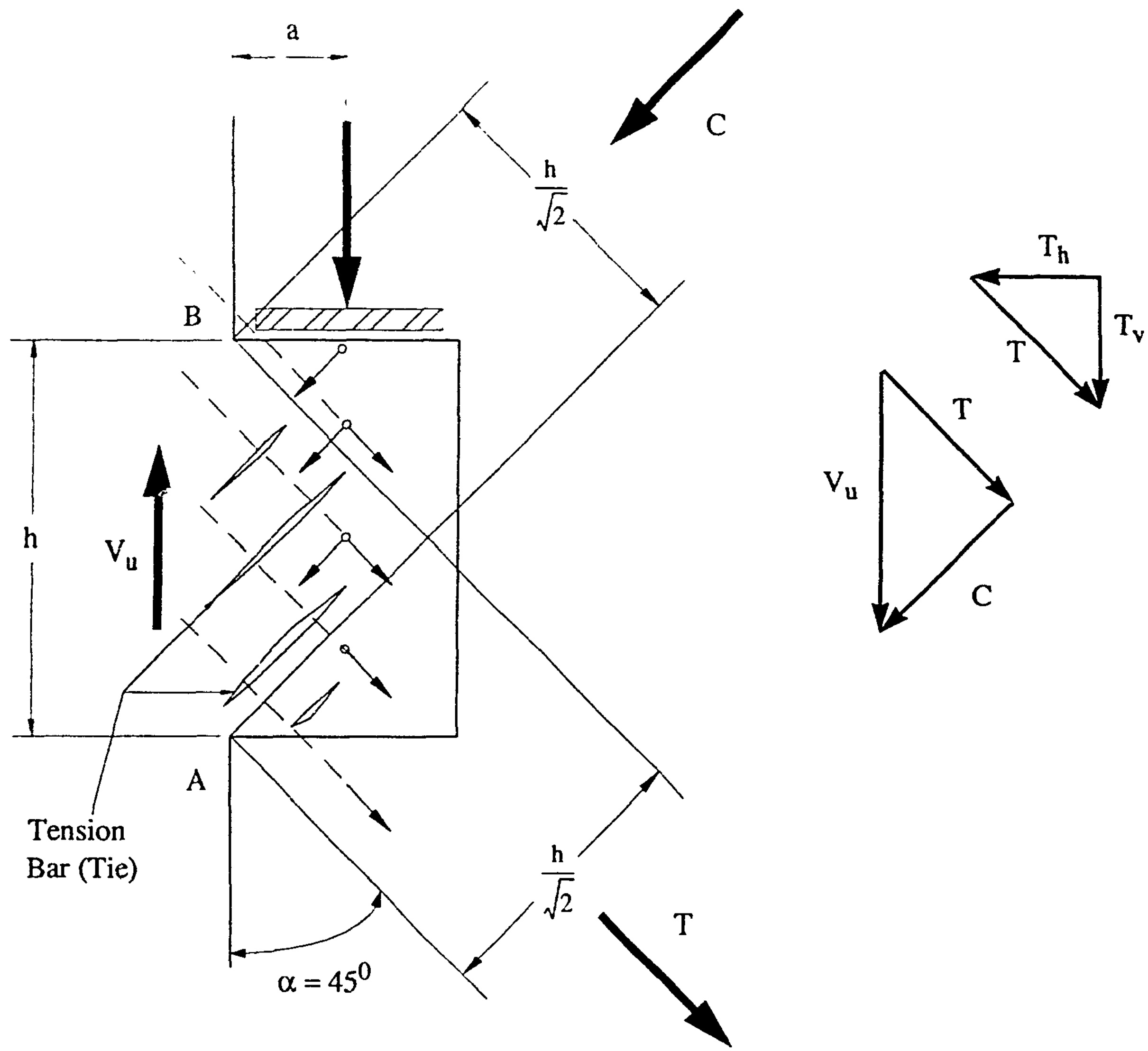


FIGURE 4.1 Strength model for horizontally reinforced



$$V_u = \frac{C + T}{\sqrt{2}}$$

Since $C = -T$

$$V_u = \sqrt{2}C = \sqrt{2}T$$

also

$$V_u = 2T_h = 2T_v$$

FIGURE 4.2 Strength model for Inclined reinforced concrete

TABLE 4.1 Test data for series 'S' (distributed horizontal reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
S1	3791.4 (26.1)	69000 (475.7)	21.4 (138)	35.5 (158)	34.0 (151.4)	64.4 (286.5)	37.3 (166.0)	1.04	N/A	1660.4 (11.4)	0.44
S3	3791.4 (26.1)	69000 (475.7)	21.4 (138)	30.5 (136)	34.0 (151.4)	64.4 (286.5)	37.3 (166.0)	0.90	N/A	1429.0 (9.8)	0.38
S4	4566.0 (31.5)	40000 (275.8)	21.3 (137)	44.5 (198)	40.9 (182.0)	37.3 (166.0)	37.3 (166.0)	N/A	1.19	2089.6 (14.4)	0.46
S5	4566.0 (31.5)	40000 (275.8)	21.4 (138)	44.4 (198)	40.9 (182)	37.3 (166.0)	37.3 (166.0)	N/A	1.19	2079.6 (14.3)	0.46
average	4178.7 (28.8)	54.5 (375.8)	21.38 (137.8)	38.73 (172.2)	37.4 (166.7)	50.8 (226.2)	37.3 (166.0)	0.97	1.19	1814.6 (12.5)	0.44

TABLE 4.2 Test data for series 'H' (grouped horizontal reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
H1	3346.3 (23.1)	40000 (275.8)	21.8 (140)	50.2 (233)	30.6 (136.1)	37.3 (166.0)	37.3 (166.0)	1.64	N/A	2310.3 (15.9)	0.69
H2	3346.3 (23.1)	40000 (275.8)	21.8 (140)	51.9 (231)	30.6 (136.1)	37.3 (166.0)	37.3 (166.0)	1.70	N/A	2386.2 (16.4)	0.71
H3	3346.3 (23.1)	40000 (275.8)	21.8 (140)	47.3 (210)	30.6 (136.1)	37.3 (166.0)	37.3 (166.0)	1.54	N/A	2175.0 (15.0)	0.65
average	3346.3 (23.1)	40000 (275.8)	21.8 (140)	49.8 (224.7)	30.6 (136.1)	37.3 (166.0)	37.3 (166.0)	1.63	N/A	2290.4 (15.8)	0.68

TABLE 4.3 Test data for series 'R' (varying depth corbel with distributed horizontal reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
R1	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	30.5 (135.6)	23.4 (104.3)	55.8 (248.2)	37.2 (165.5)	1.30	NA	1452.4 (10.0)	0.55
R2	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	30.0 (133.4)	23.4 (104.3)	55.8 (248.2)	37.2 (165.5)	1.28	NA	1428.6 (9.8)	0.54
R3	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	28.5 (126.8)	23.4 (104.3)	55.8 (248.2)	37.2 (165.5)	1.21	NA	1357.1 (9.4)	0.51
average	2634.0 (18.2)	60000 (414.0)	21.0 (135.5)	29.7 (131.9)	23.4 (104.3)	55.8 (248.2)	37.2 (165.5)	1.26	NA	1412.7 (9.7)	0.53

TABLE 4.4 Test data from series 'Q' (varying depth corbel with grouped horizontal reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
Q1	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	33.5 (149.0)	28.9 (128.9)	55.8 (248.2)	37.2 (165.5)	1.16	NA	1595.2 (11.0)	0.49
Q2	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	30.4 (135.2)	28.9 (128.9)	55.8 (248.2)	37.2 (165.5)	1.05	NA	1447.6 (9.9)	0.44
Q3	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	34.8 (154.8)	28.9 (128.9)	55.8 (248.2)	37.2 (165.5)	1.20	NA	1657.1 (11.4)	0.51
average	3255.5 (22.5)	60000 (414.0)	21.0 (135.5)	32.9 (146.3)	28.9 (128.9)	55.8 (248.2)	37.2 (165.5)	1.14	NA	1566.6 (10.8)	0.48

TABLE 4.5 Test data from series 'T' (grouped and distributed horizontal reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
T1	1176.0 (8.1)	60000 (414.0)	21.0 (135.5)	13.75 (61.2)	10.5 (46.6)	55.8 (165.9)	37.3 (165.9)	1.31	N/A	654.8 (4.51)	0.56
T2	1176.0 (8.1)	60000 (414.0)	21.0 (135.5)	16.25 (72.3)	10.5 (46.6)	55.8 (165.9)	37.3 (165.9)	1.55	N/A	773.8 (5.33)	0.66
average	1176.0 (8.1)	60000 (414.0)	21.0 (135.5)	15.0 (66.72)	10.5 (46.6)	55.8 (165.9)	37.3 (165.9)	1.43	N/A	714.3 (4.92)	0.61

TABLE 4.6 Test data for series 'C' (distributed horizontal with vertical reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
C1	3346.3 (23.1)	69000 (475.7)	21.9 (132.5)	29.8 (132.5)	35.5 (157.9)	64.4 (286.5)	37.3 (166.0)	0.84	N/A	1357.6 (9.4)	0.35
C2	3346.3 (23.1)	69000 (475.7)	21.6 (140)	35.2 (156.6)	35.0 (155.7)	64.4 (286.5)	37.3 (166.0)	1.00	N/A	1626.6 (11.2)	0.42
C3	3346.3 (23.1)	69000 (475.7)	21.6 (140)	39.6 (176.4)	35.0 (155.7)	64.4 (286.5)	37.3 (166.0)	1.13	N/A	1832.6 (12.6)	0.48
average	3346.3 (23.1)	69000 (475.7)	21.7 (137.5)	34.9 (155.2)	35.2 (155.8)	64.4 (286.5)	37.3 (166.0)	0.99	N/A	1605.6 (11.1)	0.42

TABLE 4.7 Test data for series 'V' (grouped horizontal with vertical reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
V1	3607.7 (24.9)	40000 (275.8)	21.7 (140)	51.5 (229.1)	33.0 (146.8)	37.3 (166.0)	37.3 (166.0)	1.56	N/A	2367.8 (16.3)	0.66
V2	3607.7 (24.9)	40000 (275.8)	21.7 (140)	52.3 (232.6)	33.0 (146.8)	37.3 (166.0)	37.3 (166.0)	1.58	N/A	2404.6 (16.6)	0.67
V3	3607.7 (24.9)	40000 (275.8)	21.7 (140)	54.5 (242.6)	33.0 (146.8)	37.3 (166.0)	37.3 (166.0)	1.65	N/A	2505.7 (17.3)	0.69
average	3607.7 (24.9)	40000 (275.8)	21.7 (140)	52.8 (234.8)	33.0 (146.8)	37.3 (166.0)	37.3 (166.0)	1.60	N/A	2426.0 (16.7)	0.67

TABLE 4.8 Test data for series 'B' (inclined reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
B2	3752.5 (25.9)	73000 (503.3)	21.6 (140)	47.5 (211.3)	69.0 (306.9)	77.5 (344.6)	42.5 (189.0)	N/A	1.12	2195.0 (15.1)	0.59
B4	4329.0 (29.8)	73000 (503.3)	21.4 (138)	61.0 (271.3)	78.7 (350.0)	77.5 (344.6)	42.5 (189.0)	N/A	1.43	2853.1 (19.7)	0.66
B5	4329.0 (29.8)	73000 (503.3)	21.3 (137)	61.0 (271.3)	78.2 (347.8)	77.5 (344.6)	42.5 (189.0)	N/A	1.43	2869.2 (19.8)	0.66
B6	2613.0 (18.0)	40000 (275.8)	22.5 (145)	44.3 (197.0)	50.0 (222.4)	42.5 (189.0)	42.5 (189.0)	N/A	1.04	1968.9 (13.6)	0.75
B7	2150.0 (14.8)	40000 (275.8)	21.0 (135)	40.0 (177.9)	46.6 (207.3)	42.5 (189.0)	42.5 (189.0)	N/A	0.94	1904.7 (13.1)	0.73
B8	2150.0 (14.8)	40000 (275.8)	21.0 (135)	46.6 (207.3)	46.6 (207.3)	42.5 (189.0)	42.5 (189.0)	N/A	0.93	1892.8 (13.0)	0.72
average	3220.6 (22.2)	56500 (389.5)	21.5 (139)	50.1 (222.7)	61.5 (273.6)	60.0 (266.8)	42.5 (189.0)	N/A	1.15	2280.6 (15.7)	0.68

TABLE 4.9 Test data for series ‘A_{78}’ and ‘B_{78}’ (inclined reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
$A2_{78}$	2150.0 (14.8)	60000 (414.0)	21.0 (136)	38.5 (171.0)	38.4 (170.8)	81.9 (364.4)	54.6 (242.9)	1.00	N/A	1833.0 (12.6)	0.85
$A3_{78}$	2150.0 (14.8)	60000 (414.0)	21.0 (136)	34.0 (151.0)	38.4 (170.8)	52.9 (235.3)	35.3 (157.0)	N/A	0.96	1619.0 (11.2)	0.75
average	2150.0 (14.8)	60000 (414.0)	21.0 (136.0)	36.2 (161.0)	38.4 (170.8)	67.4 (299.8)	44.9 (200.)	1.00	0.96	1726.0 (11.9)	0.80
$B4_{78}$	4846.0 (33.4)	60000 (414.0)	21.0 (136)	45.0 (200.1)	86.5 (384.8)	81.9 (364.4)	54.6 (242.9)	N/A	0.82	2140.0 (14.7)	0.44
$B5_{78}$	4846.0 (33.4)	60000 (414.0)	21.0 (136)	45.0 (200.1)	86.5 (384.8)	81.9 (364.4)	54.6 (242.9)	N/A	0.82	2140.0 (14.7)	0.44
$B6_{78}$	4846.0 (33.4)	60000 (414.0)	21.0 (136)	45.0 (200.1)	86.5 (384.8)	52.9 (235.3)	35.3 (157.0)	N/A	1.27	2140.0 (14.7)	0.44
average	4846.0 (33.4)	60000 (414.0)	21.0 (136)	45.0 (200.1)	86.5 (384.8)	72.2 (321.4)	48.2 (214.3)	N/A	0.97	2140.0 (14.7)	0.44

TABLE 4.10 Test data from series 'P' (varying depth corbel with inclined reinforcement)

Sample	f'_c psi (MPa)	f_y psi (MPa)	bh in^2 (cm^2)	V_u kips (kN)	V_{uc} based on calculated concrete capacity kips (kN)	V_{us} based on calculated steel capacity kips (kN)	V_{up} based on f_y=40000 psi kips (kN)	V_u/V_{uc}	$V_u/V_{us(p)}$	$v_u=V_u/bh$ psi (MPa)	v_u/f'_c
1	2	3	4	5	6c	6s	6p	7=5/6c	8=5/6s(p)	9=5/4	10=7/2
P1	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	26.0 (115.6)	51.5 (229.1)	71.2 (316.7)	47.4 (211.2)	N/A	0.55	1238.1 (8.54)	0.43
P2	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	30.5 (135.6)	51.5 (229.1)	71.2 (316.7)	47.4 (211.2)	N/A	0.64	1452.4 (10.0)	0.50
P3	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	32.5 (144.6)	51.5 (229.1)	71.2 (316.7)	47.4 (211.2)	N/A	0.68	1547.6 (10.7)	0.54
average	2884.8 (19.9)	60000 (414.0)	21.0 (135.5)	29.7 (132.0)	51.5 (229.1)	71.2 (316.7)	47.4 (211.2)	N/A	0.62	1412.7 (9.74)	0.49

CHAPTER V

SEGMENTED CONSTRUCTION

5.1 General

The formulas presented so far can be used for segmented construction, providing that the friction coefficient, μ, is introduced and that anchoring of reinforcement is assured. Normally the friction coefficient $\mu<1$, as the value of $\mu=1$ applies to monolithic construction. The formulas already presented earlier are now introduced with the friction coefficient.

5.2 For Horizontal Reinforcement

In terms of steel capacity

$$V_u = \mu A_{hf} f_y \qquad [54]$$

$$v_u = \mu \rho_{hf} f_y \qquad [55]$$

In terms of concrete capacity

$$V_u = \mu 0.424 f'_c bh \qquad [56]$$

$$v_u = \mu 0.424 f'_c \qquad [57]$$

5.3 For Inclined Reinforcement

In terms of steel capacity

$$V_u = \mu \sqrt{2} A_{wf} f_y \qquad [58]$$

$$v_u = \mu \rho_{wf} f_y \qquad [59]$$

In terms of concrete capacity

$$V_u = \mu 0.85f'_c bh \quad [60]$$

$$v_u = \mu 0.85f'_c \quad [61]$$

For the case where the coefficient of friction is close to zero, $\mu \approx 0$, only the steel can support. The free body diagram corresponding to such a case is shown in Figures 5.1 and 5.2. As the tensile force is developed in the reinforcement, the horizontal component "clamps" the concrete together. The concrete capacity must be checked in order to avoid failure in compression. Shear resistance will be provided by the vertical component in the reinforcement. The total shear resistance will be governed by the lower capacity of either steel or concrete, provided that adequate reinforcement is anchored.

Figure 5.2 illustrates the use of inclined reinforcement against a smooth contact surface. The compression strength of the concrete as defined earlier is $V_u=C=0.85f'_c bh$. Shear capacity provided by the steel is:

$$V_u = \frac{T}{\sqrt{2}} = \frac{A_{wf} f_y}{\sqrt{2}} \quad [62]$$

The amount of reinforcement required is:

$$A_{wf} = \frac{\sqrt{2}V}{f_y} \qquad \text{but} \qquad V \leq 0.85f'_c bh \quad [63]$$

In the case of horizontal reinforcement (Fig. 5.1), it is accepted that the horizontal steel is under direct shear (T_v) as well as tension (T_h). Hence the allowable stress, f_s, of steel should be reduced to $f_s=0.5f_y$. From the free body diagram it can be seen that:

$$V_u = A_{hf} f_s = 0.5 A_{hf} f_y \quad [64]$$

or in terms of the concrete capacity, $V_u = 0.85f'_c bh$. The amount of reinforcement required would then be defined as:

$$A_{hf} = \frac{2V}{f_y} \qquad \text{but} \qquad V \leq 0.85 f'_c bh \qquad [65]$$

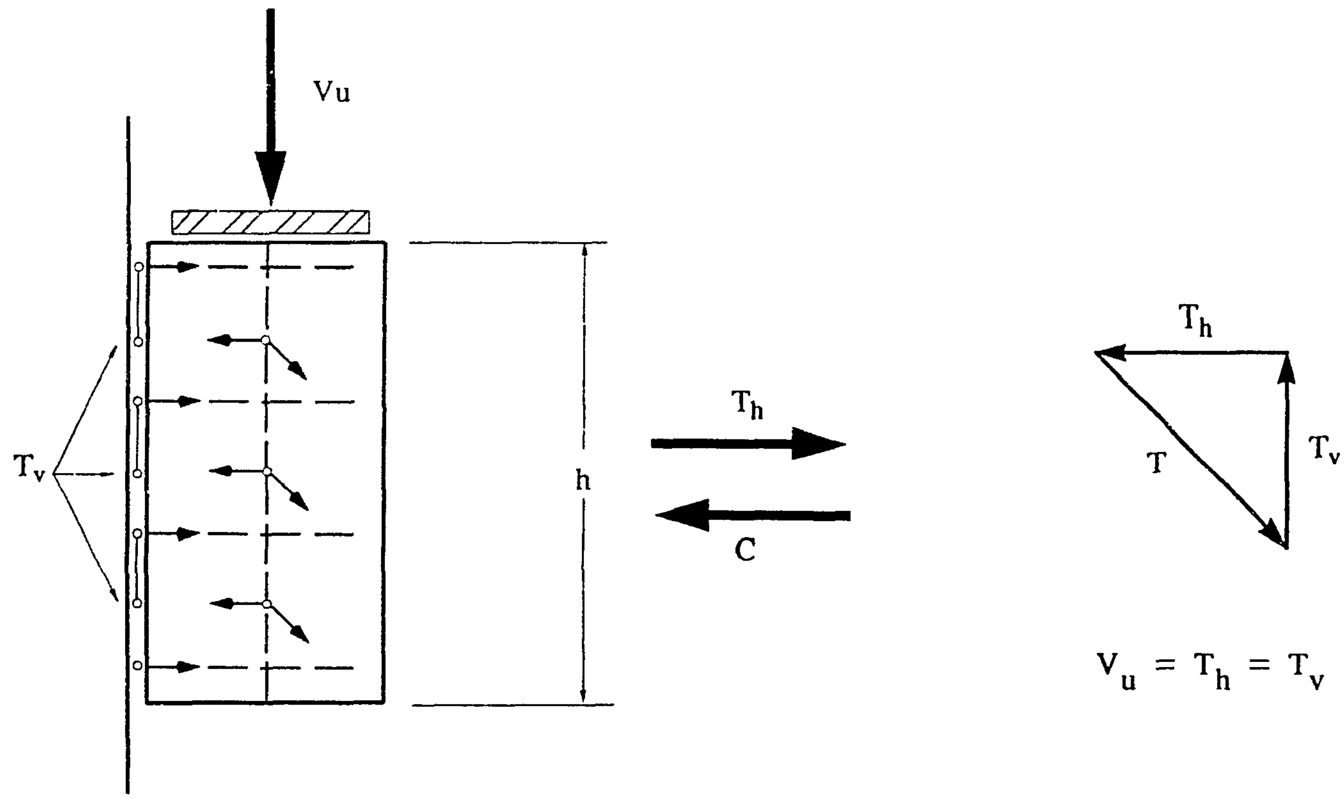
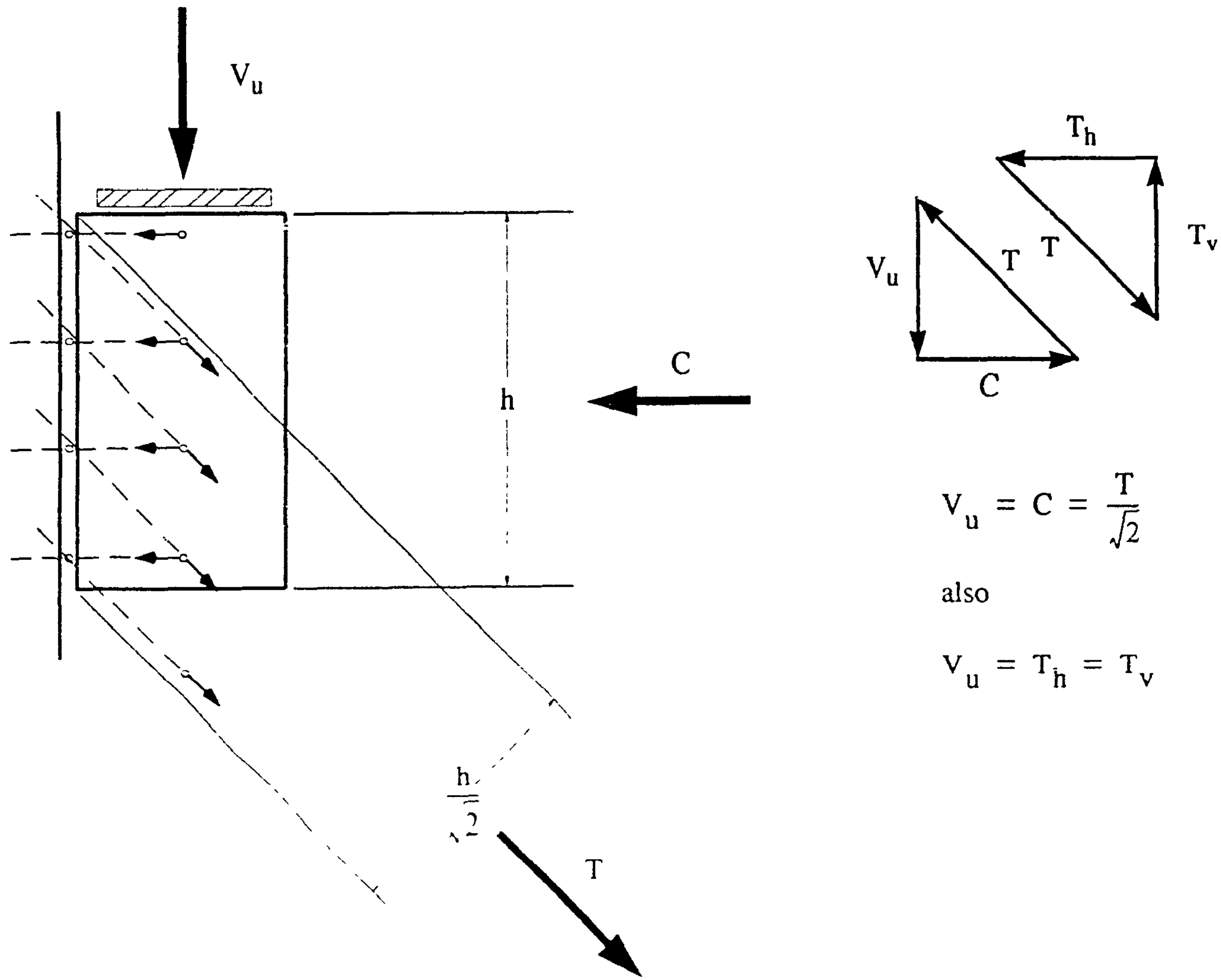


FIGURE 5.1 Model for a detached corbel with horizontal reinforcement against a smooth surface



$$V_u = C = \frac{T}{\sqrt{2}}$$

also

$$V_u = T_h = T_v$$

FIGURE 5.2 Model for a detached corbel with inclined reinforcement against a smooth surface

CHAPTER VI

DESIGN CRITERIA

6.1 General

For design practice the above derived equations should be modified to include the capacity reduction and load or safety factors as required by the respective American and Canadian codes. The American code uses a factored load = 1.4D +1.7L with an overall capacity reduction factor of $\phi = 0.85$ for shear. The Canadian code uses a factored load = 1.25D + 1.5L, while concrete and steel capacities are reduced by factors of $\phi_c = 0.60$ and $\phi_s = 0.85$ respectively.

6.2 Design Procedure

The equations derived for both horizontal and inclined reinforced concrete elements are based on the balanced condition, i.e. when the tension capacity of the reinforcement equals the compression capacity of the concrete. Designing for the balanced condition may lead to sudden failure with little warning. It is recommended to design for a maximum steel ratio of $\rho=0.75\rho_{max}$. To facilitate design, tables have been prepared for the usage of both horizontal and inclined reinforcement. Steel yield strengths of 40000 psi (275.8 MPa) and 400 MPa (58000 psi) have been considered. The following is a suggested design procedure when using the provided design Tables 6.1 to 6.4.

A. Factor loads

B. Select concrete strength and steel yield strength

C. Assume a percent ρ (suggest maximum value of $\rho=0.75\rho_{max}$)

D From Tables 6.1 to 6.4 determine "K" and corresponding ρ

Where "K" is equal to:

$$K = 0.424f'_c \times \frac{\rho}{\rho_{max}} \qquad \text{horizontal reinforcement}$$

$$K = 0.85f'_c \times \frac{\rho}{\rho_{max}} \qquad \text{inclined reinforcement}$$

E. Determine required concrete area (Factored Load/K)

F. Determine steel area (ρ x concrete area)/(steel capacity reduction or shear factor)

G. Determine concrete area with appropriate factors (area/ϕ_c)

H. Calculate new steel ratio and check to see $\rho_{min}<\rho<\rho_{max}$

I. Select section properties (check to see $a/h \leq 1$)

J. Select and place reinforcement ensuring proper covers and anchorage

Analysis and design can also be done by directly using the equations derived. The moment capacity can be checked by calculating the concrete and steel horizontal component and multiplying one of them by the lever arm. Two design examples are provided in Appendix C.

TABLE 6.1 Design Table for Horizontally Reinforced Concrete Corbels (f_y=400 MPa)

f'_c	20 MPa	25 MPa	30 MPa	35 MPa	40 MPa
	ρ [K] MPa	ρ [K] MPa	ρ [K]MPa	ρ [K] MPa	ρ [K] MPa
ρ(100%) max	0.0212 [8.48]	0.0265 [10.6]	0.0318 [12.72]	0.0371 [14.84]	0.0424 [16.96]
ρ(90%)	0.01908 [7.632]	0.02385 [9.54]	0.02862 [11.448]	0.03339 [13.356]	0.03816 [15.264]
ρ(80%)	0.01696 [6.784]	0.0212 [8.48]	0.02544 [10.176]	0.02968 [11.872]	0.03392 [13.568]
ρ(75%)	0.0159 [6.36]	0.0198 [7.95]	0.0238 [9.54]	0.0278 [11.13]	0.0318 [12.72]
ρ(70%)	0.01484 [5.936]	0.01855 [7.42]	0.02226 [8.904]	0.02597 [10.388]	0.02968 [11.872]
ρ(60%)	0.01272 [5.088]	0.0159 [6.36]	0.01908 [7.632]	0.02226 [8.904]	0.02544 [10.176]
ρ(50%)	0.0106 [4.24]	0.01325 [5.3]	0.0159 [6.36]	0.01855 [7.42]	0.0212 [8.48]
ρ(40%)	0.00848 [3.392]	0.0106 [4.24]	0.01272 [5.088]	0.01484 [5.936]	0.01696 [6.784]
ρ(30%)	0.00636 [2.544]	0.00795 [3.18]	0.00954 [3.816]	0.01113 [4.452]	0.01272 [5.088]
ρ(20%)					0.00848 [3.392]
ρ(min)	0.00579 [2.316]	0.00647 [2.59]	0.00709 [2.84]	0.00766 [3.065]	0.00819 [3.27]

TABLE 6.2 Design Table for Horizontally Reinforced Concrete Corbels (f_y=40000 psi)

f'_c	3000 psi	3500 psi	4000 psi	5000 psi	6000 psi
	ρ [K] psi	ρ [K] psi	ρ [K]psi	ρ [K] psi	ρ [K] psi
ρ(100%) max	0.0212 [848.0]	0.0265 [1060.0]	0.0318 [1272.0]	0.0371 [1484.0]	0.0424 [1696.0]
ρ(90%)	0.01908 [763.2]	0.02385 [954.0]	0.02862 [1144.8]	0.03339 [1335.6]	0.03816 [1526.4]
ρ(80%)	0.01696 [678.4]	0.0212 [763.2]	0.02544 [1017.6]	0.02968 [1187.2]	0.03392 [1356.8]
ρ(75%)	0.0159 [636.0]	0.0198 [795.0]	0.0238 [954.0]	0.0278 [1113.0]	0.0318 [1272.0]
ρ(70%)	0.01484 [593.6]	0.01855 [742.0]	0.02226 [890.4]	0.02597 [1038.8]	0.02968 [1187.2]
ρ(60%)	0.01272 [508.8]	0.0159 [636.0]	0.01908 [763.2]	0.02226 [890.4]	0.02544 [1017.6]
ρ(50%)	0.0106 [424.0]	0.01325 [530.0]	0.0159 [636.0]	0.01855 [742.0]	0.0212 [848.0]
ρ(40%	0.00848 [339.2]	0.0106 [424.0]	0.01272 [508.8]	0.01484 [593.6]	0.01696 [678.4]
ρ(30%)	0.00636 [254.4]	0.00795 [318.0]	0.00954 [381.6]	0.01113 [445.2]	0.01272 [508.8]
ρ(20%)					0.00848 [339.2]
ρ(min)	0.00579 [231.6]	0.00647 [258.8]	0.00709 [283.6]	0.00766 [306.4]	0.00819 [327.6]

TABLE 6.3 Design Table for Inclined Reinforced Concrete Corbels (f_y=400MPa)

f'_c	20 MPa	25 MPa	30 MPa	35 MPa	40 MPa
	ρ [K]	ρ [K]	ρ [K]	ρ [K]	ρ [K]
ρ(100%) max	0.0425 [17.0]	0.0531 [21.25]	0.06375 [25.5]	0.07437 [29.75]	0.085 [34.0]
ρ(90%)	0.03825 [15.3]	0.478 [19.12]	0.0573 [22.95]	0.0669 [26.77]	0.0765 [30.6]
ρ(80%)	0.034 [13.6]	0.0425 [17.0]	0.051 [20.4]	0.0595 [23.80]	0.068 [27.2]
ρ(75%)	0.0319 [12.75]	0.0398 [15.94]	0.0478 [19.12]	0.0558 [22.31]	0.0637 [25.50]
ρ(70%)	0.02975 [11.9]	0.0371 [14.87]	0.0446 [17.85]	0.0520 [20.82]	0.0595 [23.8]
ρ(60%)	0.0255 [10.2]	0.0318 [12.75]	0.03825 [15.3]	0.04462 [17.85]	0.051 [20.4]
ρ(50%)	0.02125 [8.5]	0.0265 [10.62]	0.0318 [12.75]	0.03718 [14.87]	0.0425 [17.0]
ρ(40%)	0.017 [6.8]	0.0212 [8.5]	0.0255 [10.2]	0.02975 [11.90]	0.034 [13.6]
ρ(30%)	0.01275 [5.1]	0.0159 [6.375]	0.0191 [7.65]	0.0223 [8.925]	0.0255 [10.2]
ρ(20%)	0.0085 [3.4]	0.0106 [4.25]	0.01275 [5.1]	0.01487 [5.95]	0.017 [6.80]
ρ(10%)					0.0085 [3.4]
ρ(min)	0.00579 [2.316]	0.00647 [2.59]	0.00709 [2.84]	0.00766 [3.065]	0.00819 [3.27]

TABLE 6.4 Design Table for Inclined Reinforced Concrete Corbels (f_y=40000psi)

f'_c	3000 psi	3500 psi	4000 psi	5000 psi	6000 psi
	ρ [K] psi	ρ [K] psi	ρ [K]psi	ρ [K] psi	ρ [K] psi
ρ(100%) max	0.0425 [1700.0]	0.0531 [2124.0]	0.06375 [2550.0]	0.07437 [2974.8]	0.085 [3400.0]
ρ(90%)	0.03825 [1530.0]	0.478 [1911.6]	0.0573 [2295.0]	0.0669 [2677.3]	0.0765 [3060.0]
ρ(80%)	0.034 [1360.0]	0.0425 [1699.2]	0.051 [2040.0]	0.0595 [2379.8]	0.068 [2720.0]
ρ(75%)	0.0319 [1275.0]	0.0398 [1593.0]	0.0478 [1912.5]	0.0558 [2231.1]	0.0637 [2550.0]
ρ(70%)	0.02975 [1190.0]	0.0371 [1486.8]	0.0446 [1785.0]	0.0520 [2082.4]	0.0595 [2380.0]
ρ(60%)	0.0255 [1020.0]	0.0318 [1274.4]	0.03825 [1530.0]	0.04462 [1784.9]	0.051 [2040.0]
ρ(50%)	0.02125 [850.0]	0.0265 [1062.0]	0.0318 [1275.0]	0.03718 [1487.4]	0.0425 [1700.0]
ρ(40%)	0.017 [680.0]	0.0212 [849.6]	0.0255 [1020.0]	0.02975 [1189.9]	0.034 [1360.0]
ρ(30%)	0.01275 [510.0]	0.0159 [637.2]	0.0191 [765.0]	0.0223 [892.4]	0.0255 [1020.0]
ρ(20%)	0.0085 [340.0]	0.0106 [424.8]	0.01275 [510.0]	0.01487 [594.9]	0.017 [680.0]
ρ(10%)					0.0085 [340.0]
ρ(min)	0.00579 [231.6]	0.00647 [258.8]	0.00709 [283.6]	0.00766 [306.44]	0.00819 [327.6]

CHAPTER VII

SUMMARY AND CONCLUSIONS

7.1 Conclusion

The following conclusions can be drawn from the results of laboratory testing and free body diagram analysis.

The free body diagrams presented here can be used safely and accurately to design or analyze corbels, deep beams or other concrete structures subjected to a predominant shear force (i.e. a shear span to overall depth ratio of $a/h \leq 1$). Ultimate shear capacity is dependent on the strength of concrete, strength of steel, the amount of reinforcement and its placement. The work models presented here, based on multiple free body diagrams composed of compression struts and tension ties, have shown to accurately describe the test results. Based on these models, it is clear that the maximum capacity is equal to the capacity of the concrete compression strut when accompanied by an equal or greater steel tension tie capacity.

Currently shear capacity for corbels and deep beams is limited to $V_n \leq 0.2f'_c b_w d$ or $v_u/f'_c \leq 0.2$ and $V_n \leq 800b_w d$ by the American code (ACI 318-83 clause 11.9.3.2.1), and to $V_r \leq 0.25f'_c A_{cv}$ or $v_u/f'_c \leq 0.25$ by the Canadian code (CAN3-A23.3-M84 clause 11.7.5). These limits are even lower if referred to by h = full beam depth instead of d = effective depth. If it is assumed that for deep beams $d \approx 0.8 - 0.9h$ than $V_n \leq 0.16 - 0.18f'_c b_w h$ and $V_r \leq 0.2 - 0.225f'_c bh$. The findings suggests that the above limits could be significantly increased for corbels, deep beams and dapped ends.

Test results and proposed free body analysis show that the maximum capacity of corbels, dapped ends, and deep beams in terms of ultimate shear force, V_u, is as high as:

- For horizontal reinforcement uniformly distributed

-in terms of concrete capacity, $V_{uc} \leq 0.424f'_c bh$

-in terms of steel capacity, $V_{us} \leq A_{hf} f_y$

- For 45^0 inclined reinforcement

-in terms of concrete capacity, $V_{uc} \leq 0.85f'_c bh$

-in terms of steel capacity, $V_{us} \leq \sqrt{2} A_{wf} f_y$

The amount of reinforcement provided must assure that $V_{us} \geq V_{uc}$.

The maximum usable amount of reinforcement for the case of horizontal reinforcement is $\rho_{hf} = (0.424f'_c)/f_y$, for inclined reinforcement the limit is $\rho_{wf} = (0.85f'_c)/f_y$. Increasing the amount of reinforcement above these limits will not increase the shear strength since the concrete capacity is the governing factor.

Vertical stirrups do not contribute to the strength of corbels, deep beams, and dapped ends when $a/h \leq 1$ given that adequate horizontal or inclined reinforcement is present and properly anchored.

Horizontally grouped reinforcement in the tensile zone provides higher values of v_u/f'_c than does uniformly distributed horizontal reinforcement.

For the case of horizontal reinforcement not uniformly distributed, but bundled in the tension (flexural) zone or combinations of horizontal and inclined reinforcement, the maximum shear capacity will be $0.424f'_c bh \leq V_{uc} \leq 0.85f'_c bh$. Further studies are needed to establish "shear" strength limits for any combination of reinforcement.

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APPENDIX A- Cracking Details

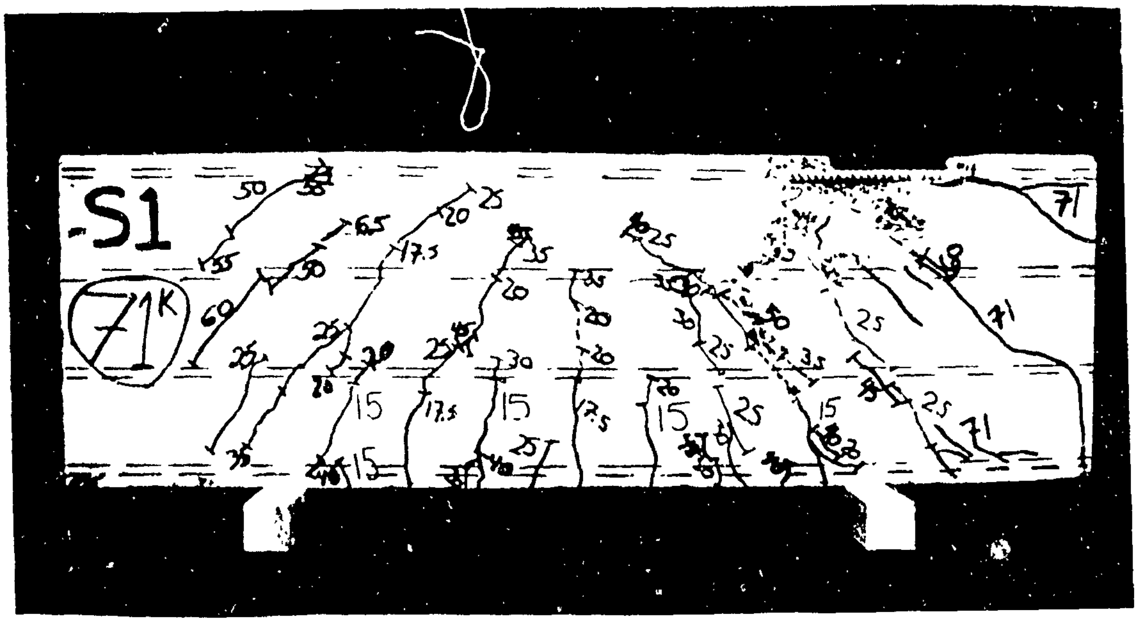


FIGURE A.1 Cracking pattern for sample S1 (failed at 35.5 kips or 158.0 kN)

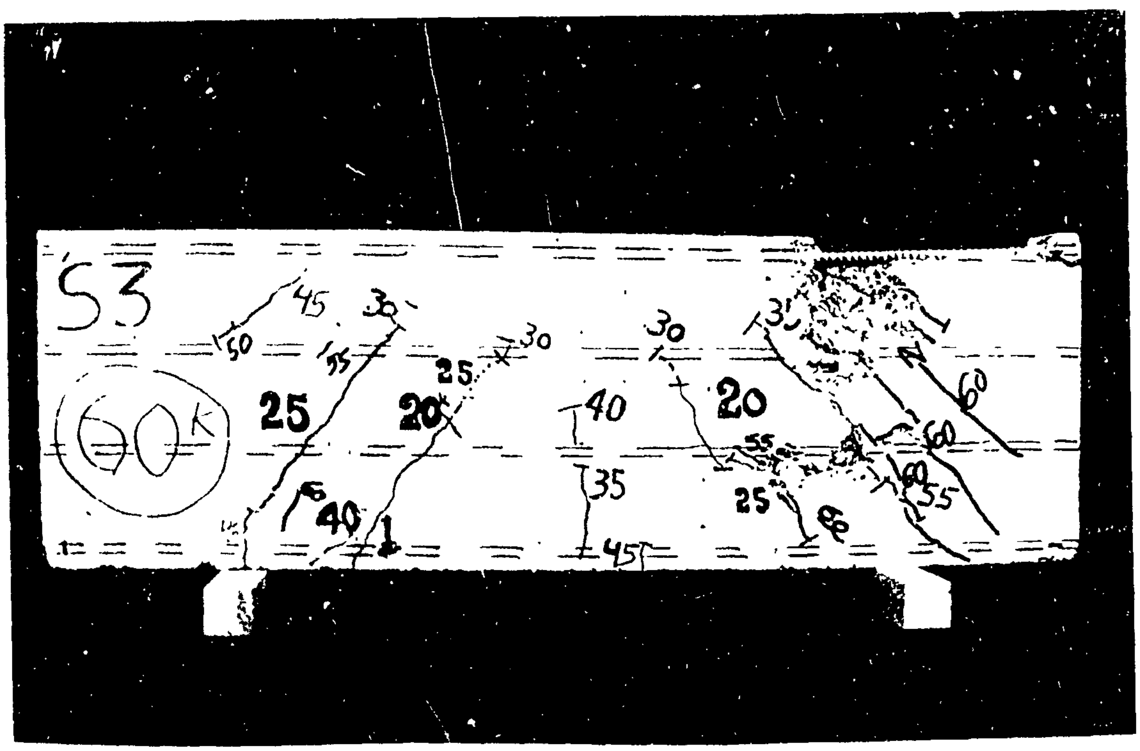


FIGURE A.2 Cracking pattern for sample S3 (failed at 30.5 kips or 136.0 kN)

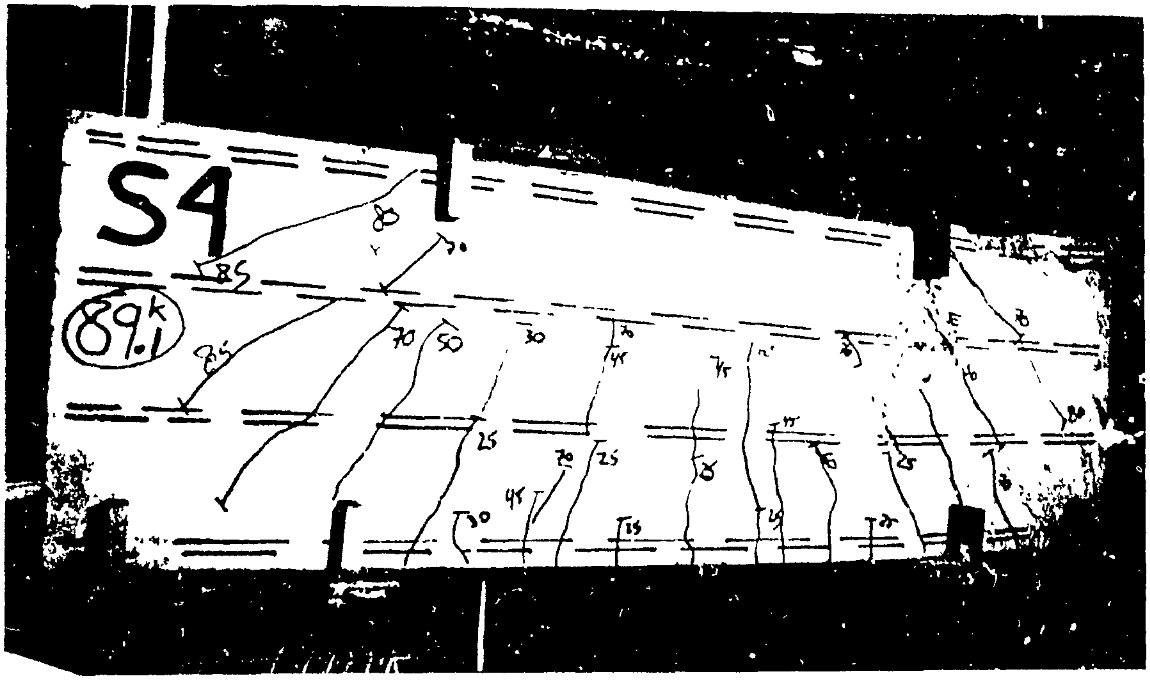


FIGURE A.3 Cracking pattern for sample S4 (failed at 44.5 kips or 198.0 kN)

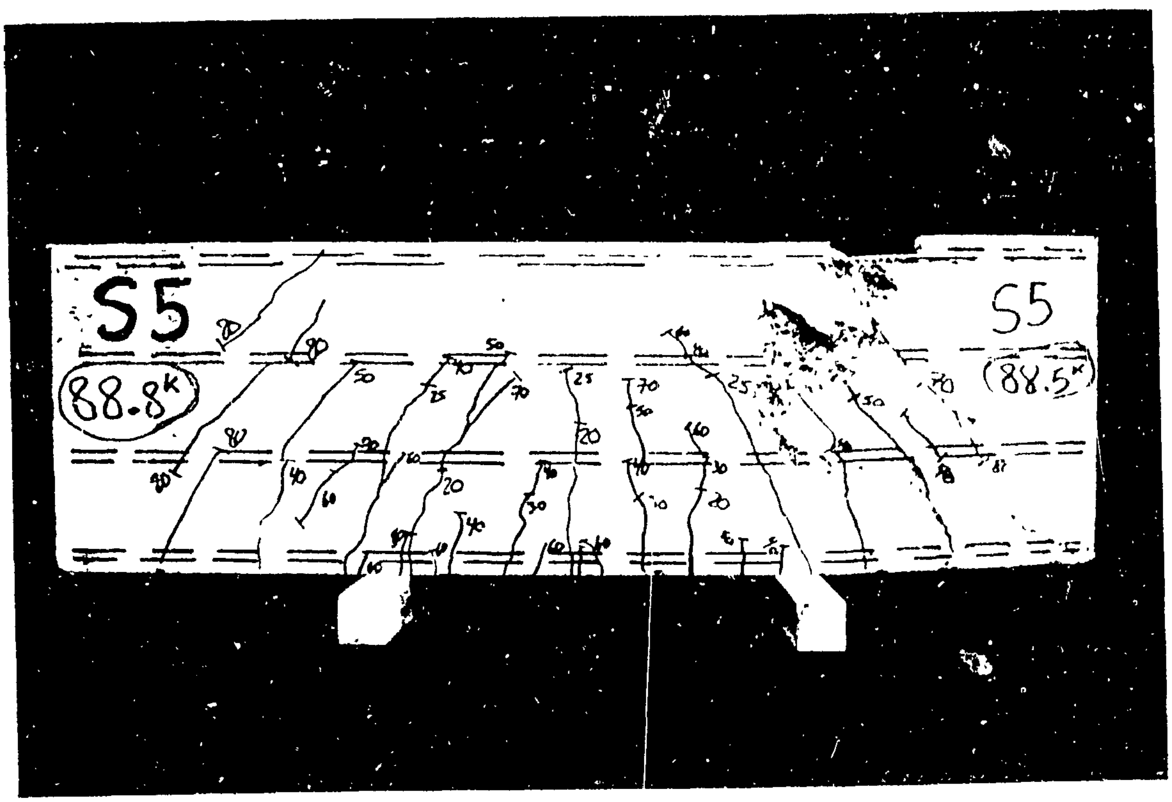


FIGURE A.4 Cracking pattern for sample S5 (failed at 44.4 kips or 198.0 kN)

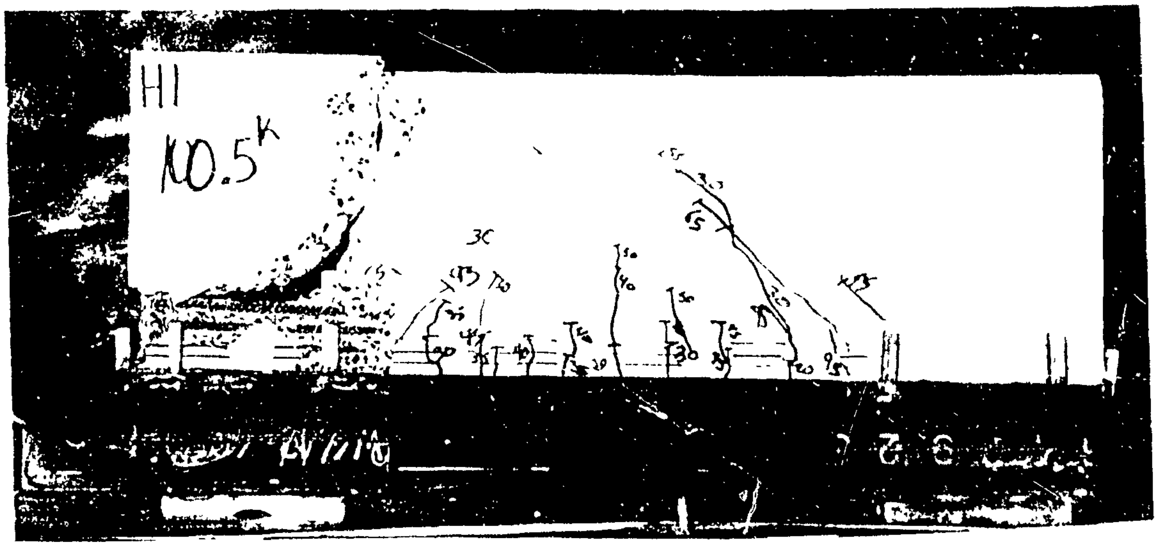


FIGURE A.5 Cracking pattern for sample H1 (failed at 50.2 kips or 233.0 kN)

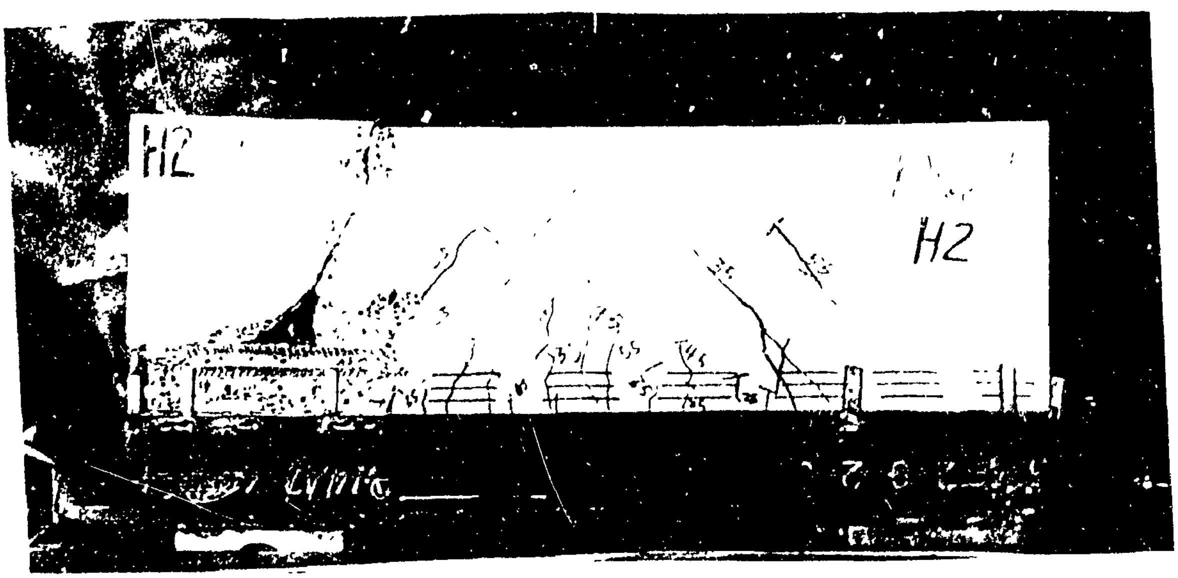


FIGURE A.6 Cracking pattern for sample H2 (failed at 51.9 kips or 231.0 kN)

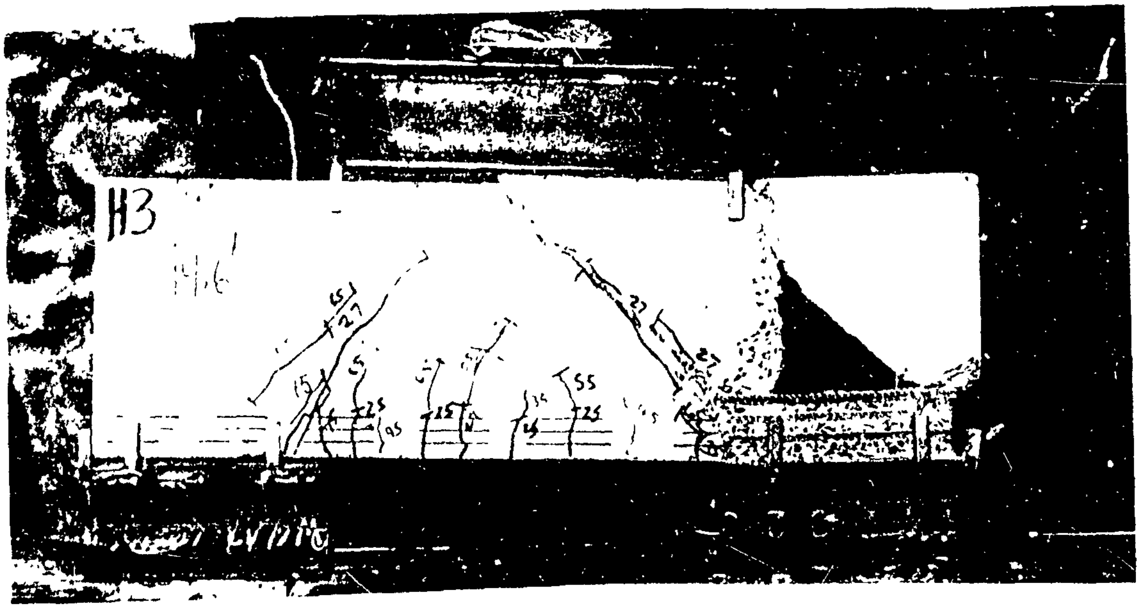


FIGURE A.7 Cracking pattern for sample H3 (failed at 47.3 kips or 210.0 kN)

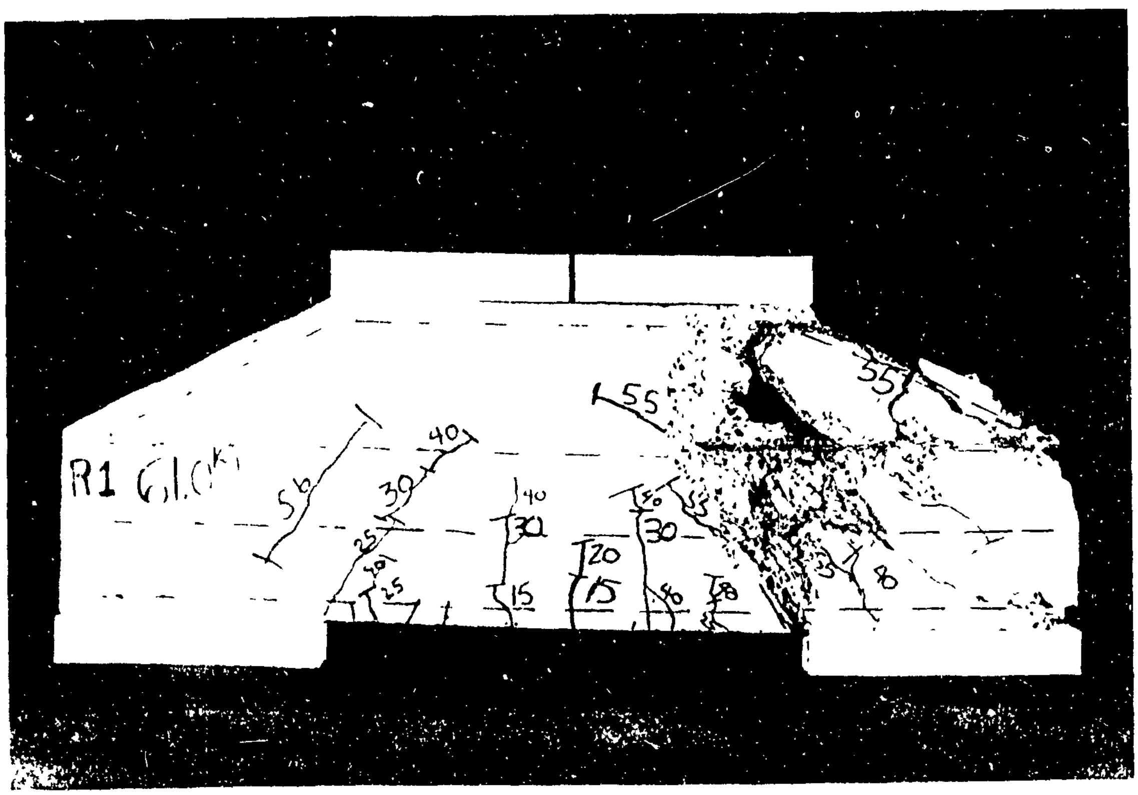


FIGURE A.8 Cracking pattern for sample R1 (failed at 30.5 kips or 135.6 kN)

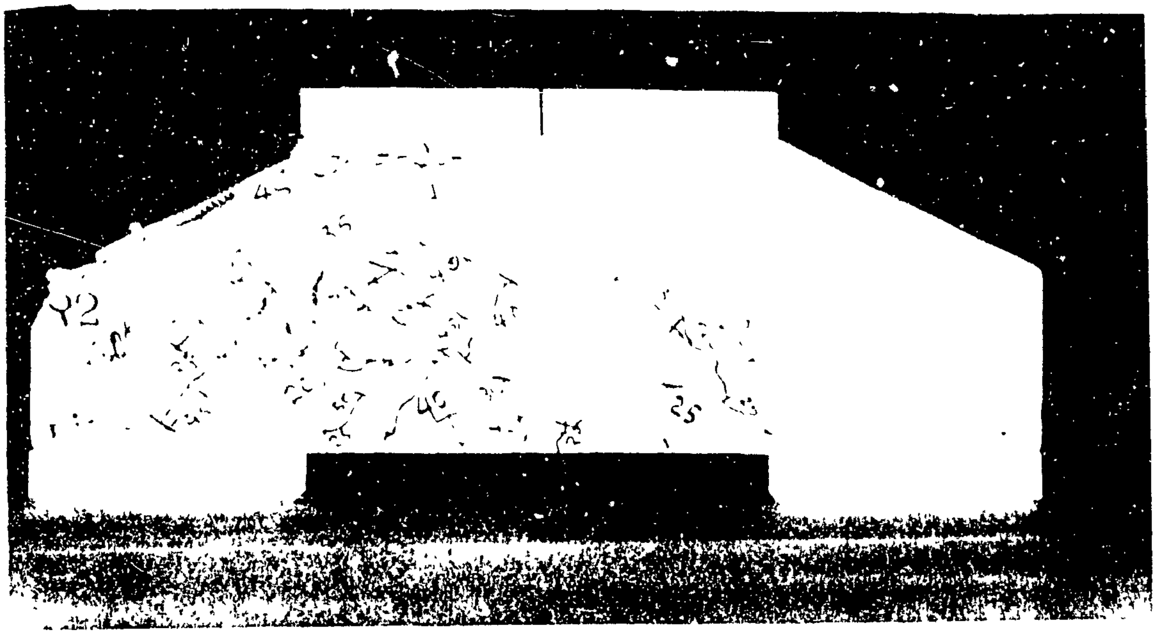


FIGURE A.9 Cracking pattern for sample R2 (failed at 30.0 kips, or 133.4 kN)

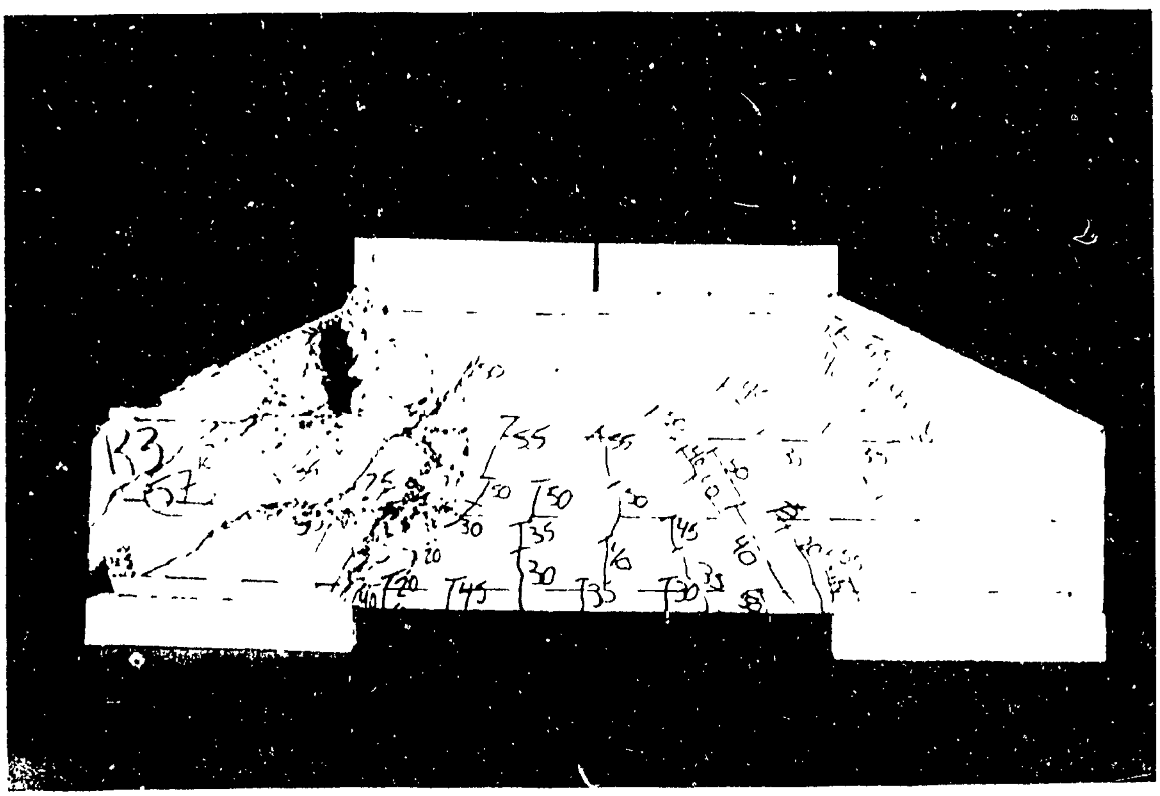


FIGURE A.10 Cracking pattern for sample R3 (failed at 28.5 kips, or 126.8 kN)

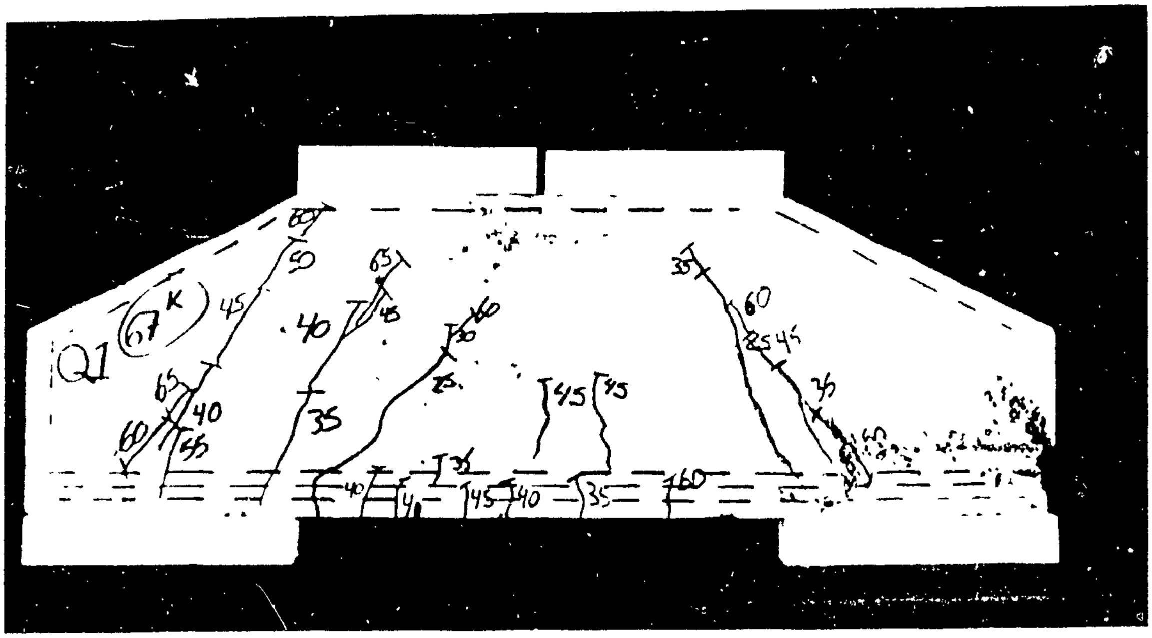


FIGURE A.11 Cracking pattern for sample Q1 (failed at 33.5 kips or 149.0 kN)

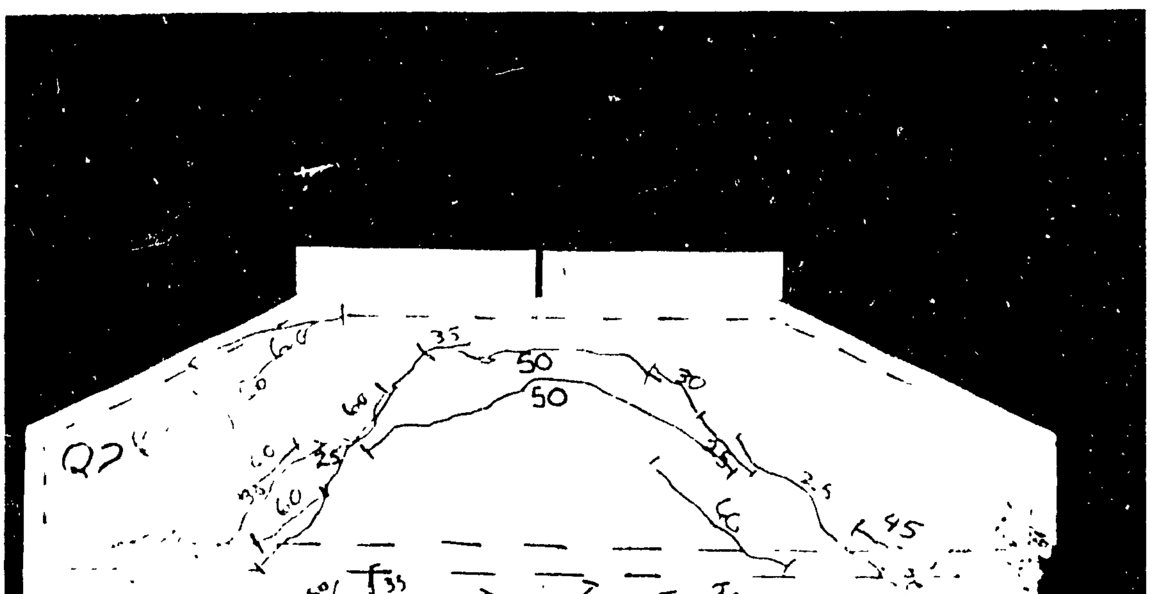


FIGURE A.12 Cracking pattern for sample Q2 (failed at 30.4 kips or 135.2 kN)

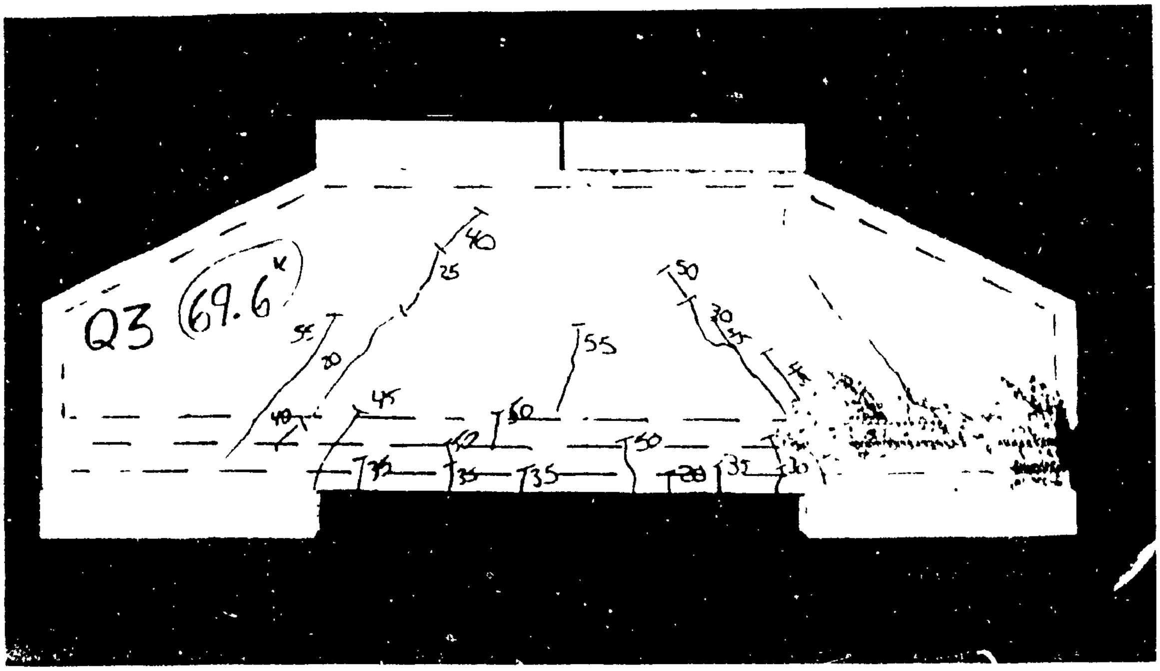


FIGURE A.13 Cracking pattern for sample Q3 (failed at 34.8 kips or 154.8 kN)

FIGURE A.14 Cracking pattern for sample T1 (failed at 13.8 kips or 61.2 kN)

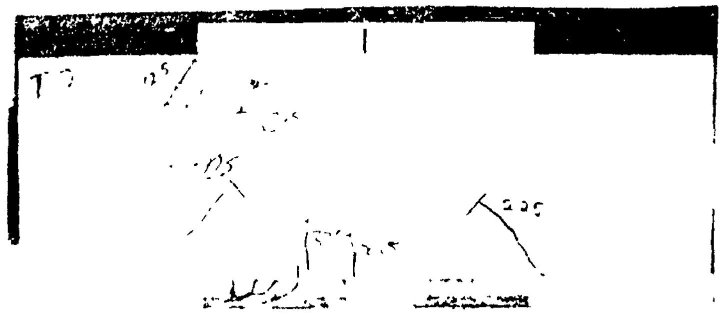


FIGURE A.15 Cracking pattern for sample T2 (failed at 16.2 kips or 72.3 kN)

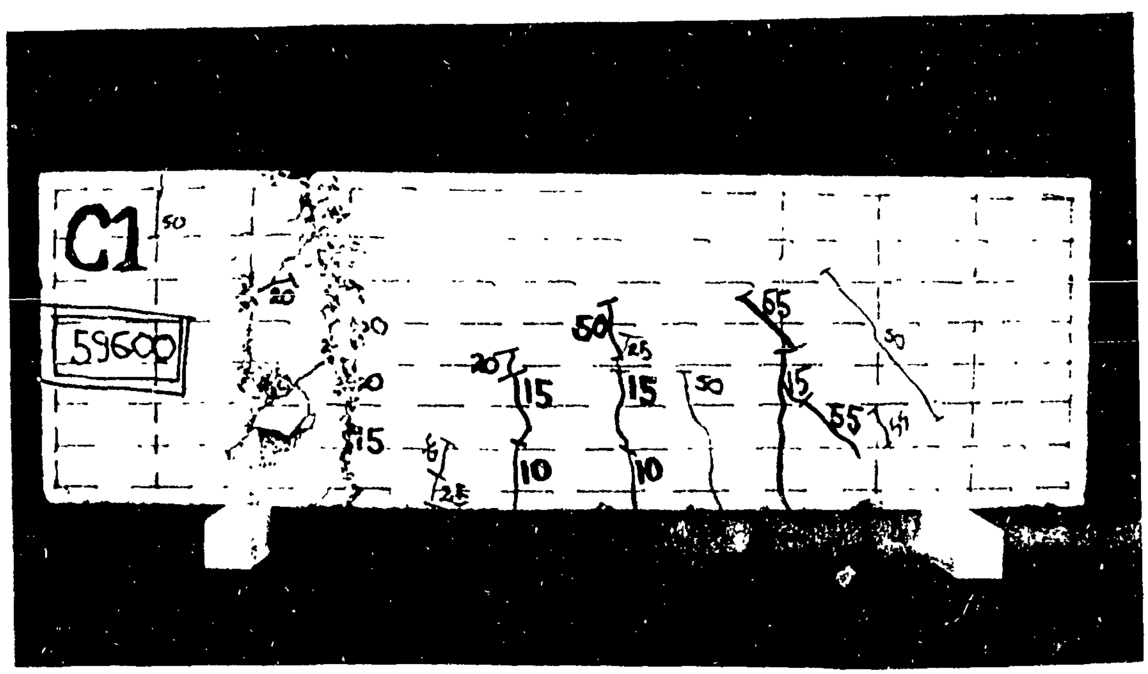


FIGURE A.16 Cracking pattern for sample C1 (failed at 29.8 kips or 132.5 kN)

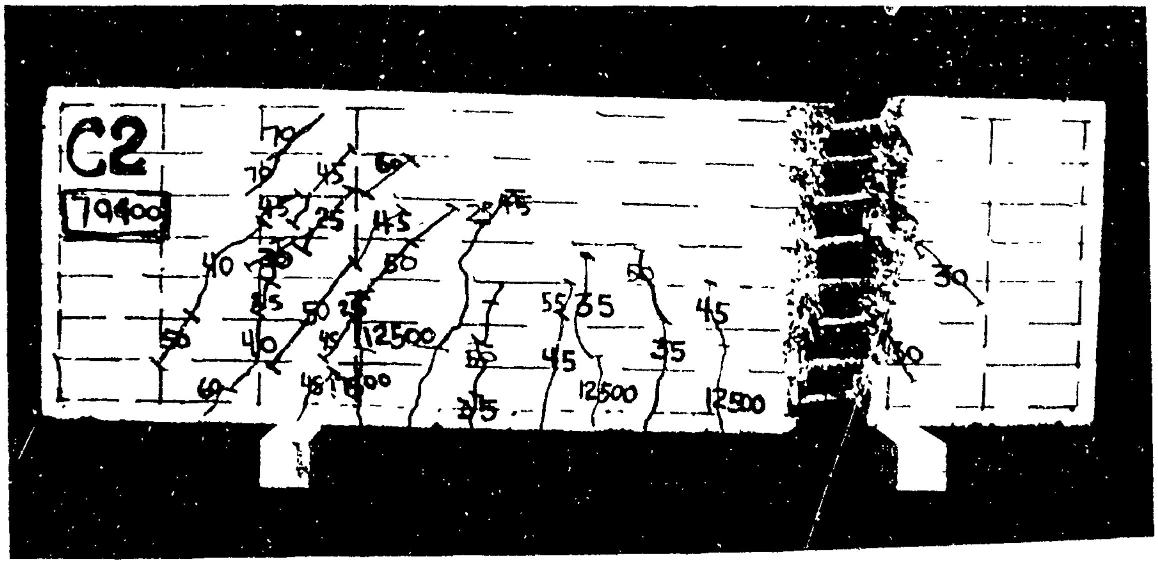


FIGURE A.17 Cracking pattern for sample C2 (failed at 35.2 kips or 156.6 kN)

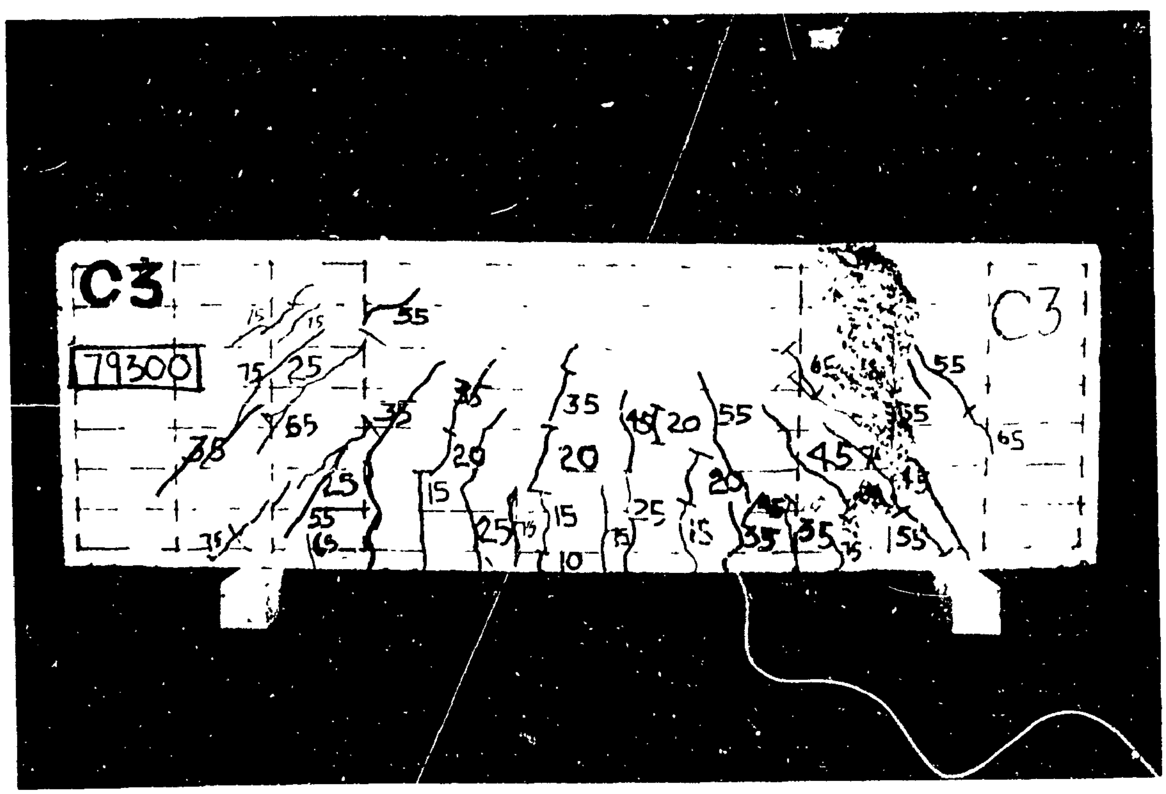


FIGURE A.18 Cracking pattern for sample C3 (failed at 39.6 kips or 176.1 kN)

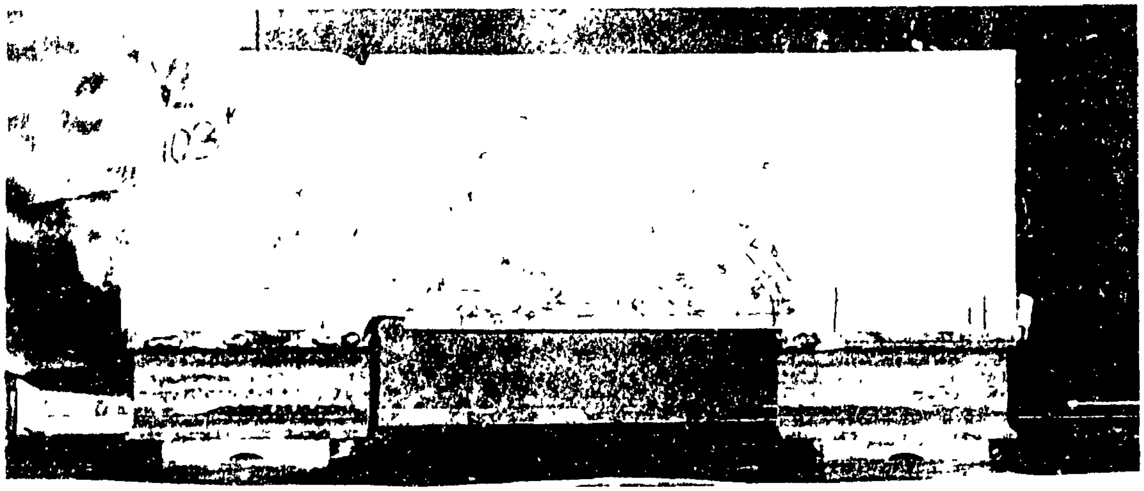


FIGURE A.19 Cracking pattern for sample V1 (failed at 51.5 kips or 229.1 kN)

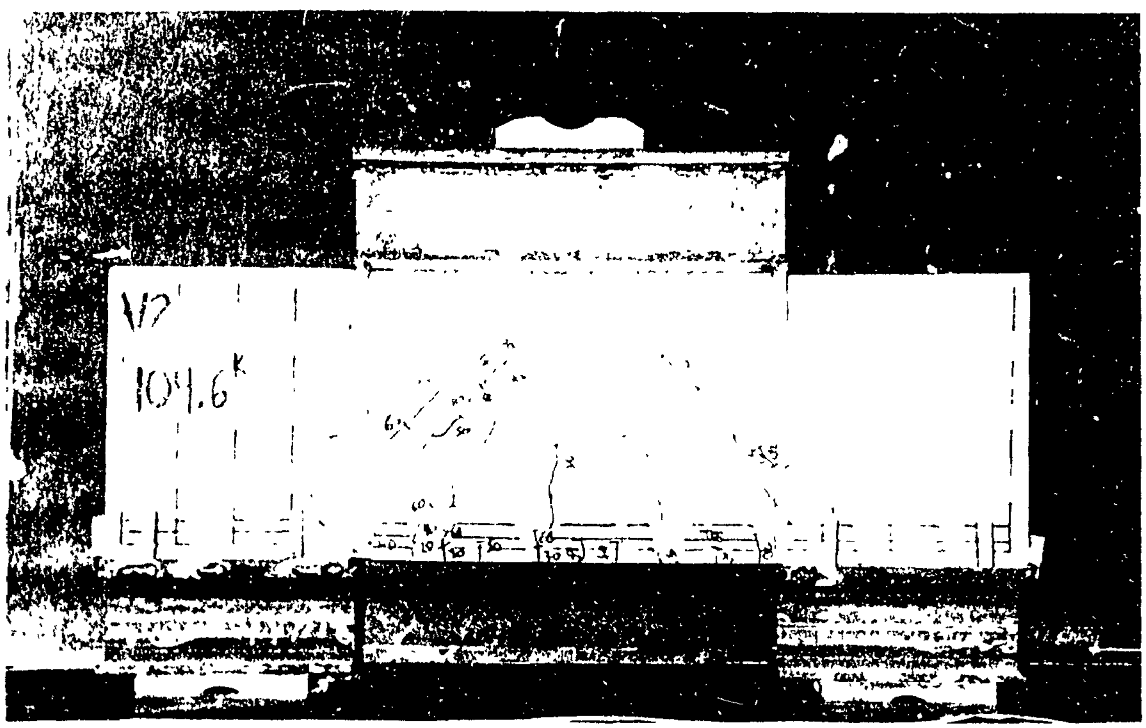


FIGURE A.20 Cracking pattern for sample V2 (failed at 52.3 kips or 232.6 kN)

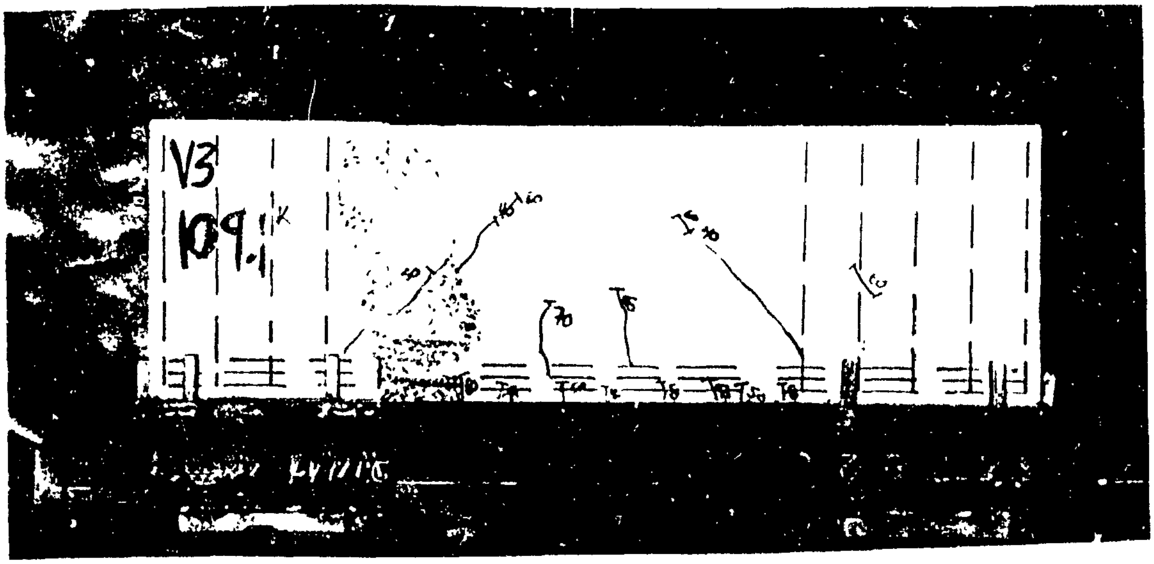


FIGURE A.21 Cracking pattern for sample V3 (failed at 54.5 kips or 242.6 kN)

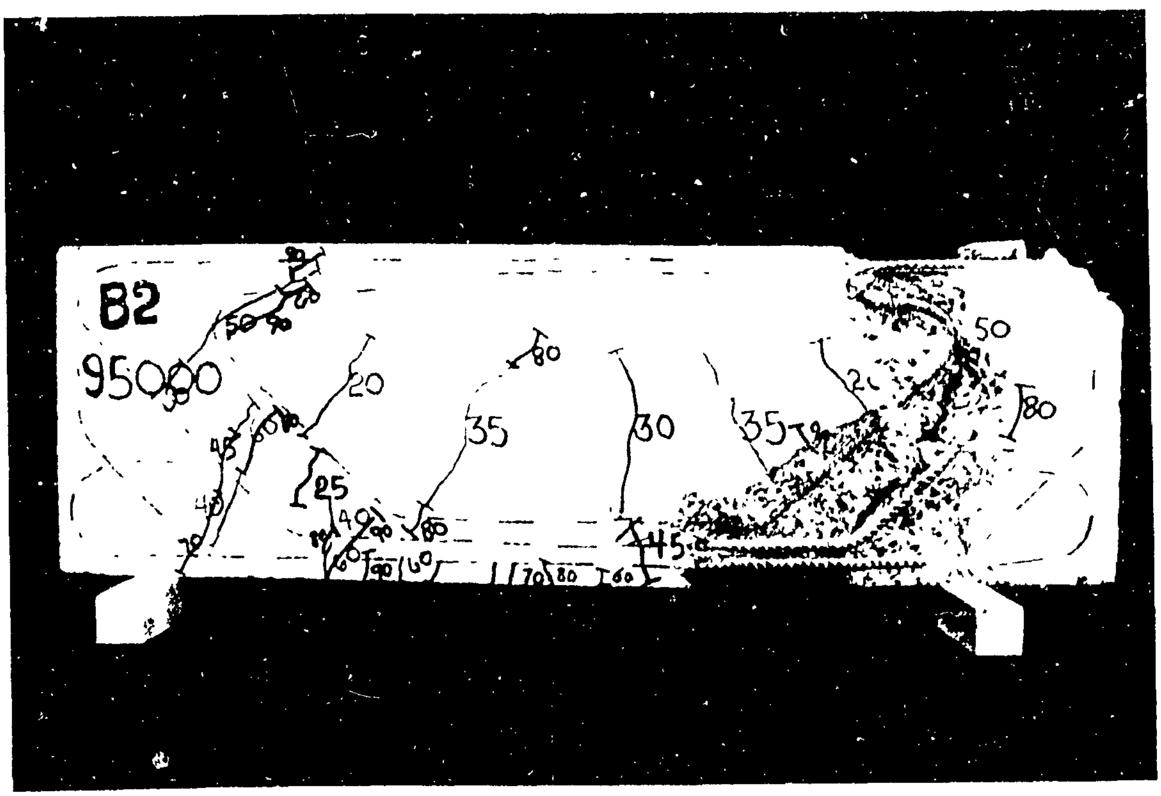


FIGURE A.22 Cracking pattern for sample B2 (failed at 47.5 kips or 211.3 kN)

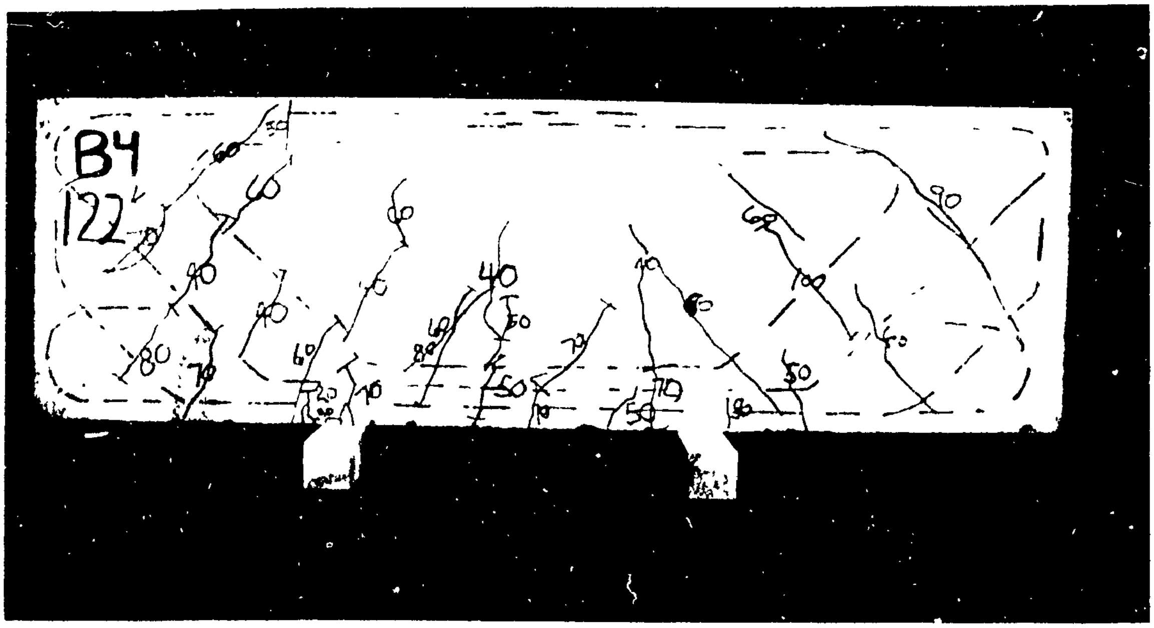


FIGURE A.23 Cracking pattern for sample B4 (failed at 61.0 kips or 271.3 kN)

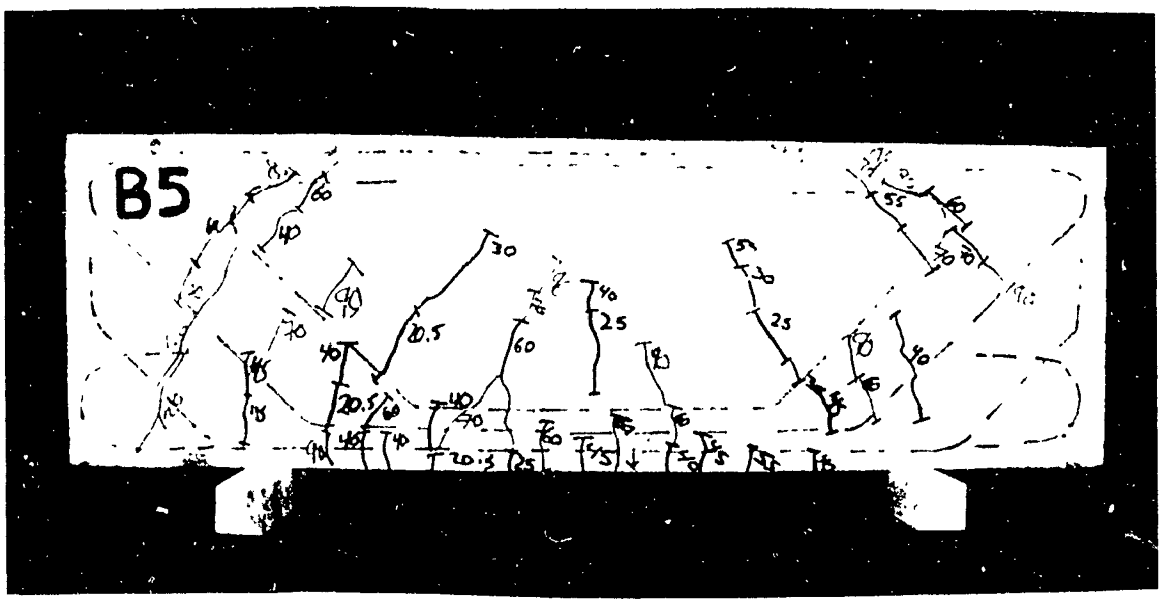


FIGURE A.24 Cracking pattern for sample B5 (failed at 61.0 kips or 271.3 kN)

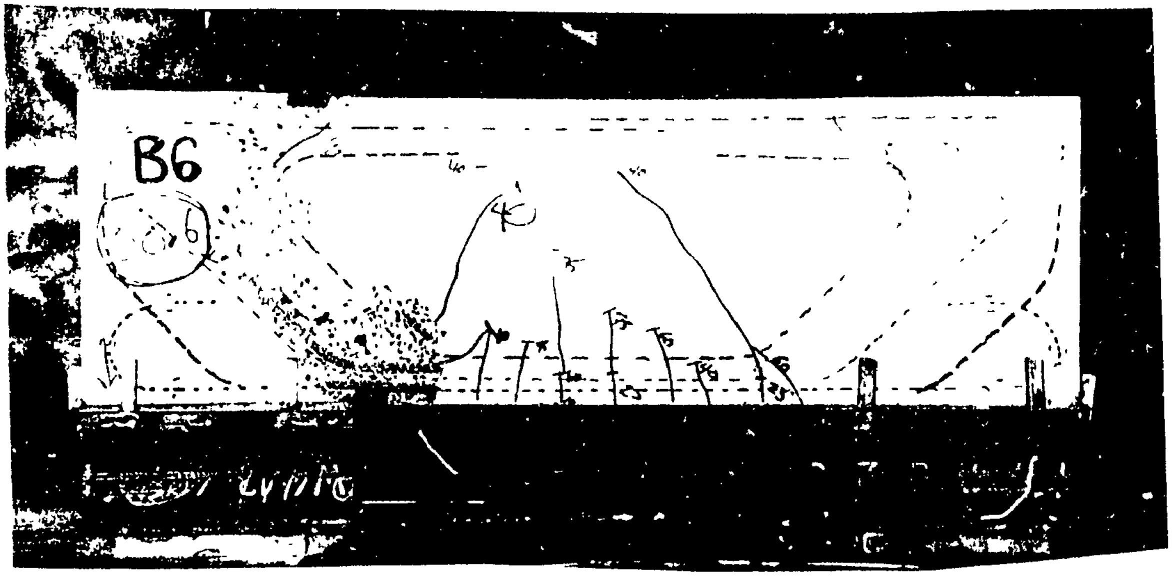


FIGURE A.25 Cracking pattern for sample B6 (failed at 44.3 kips or 197.0 kN)

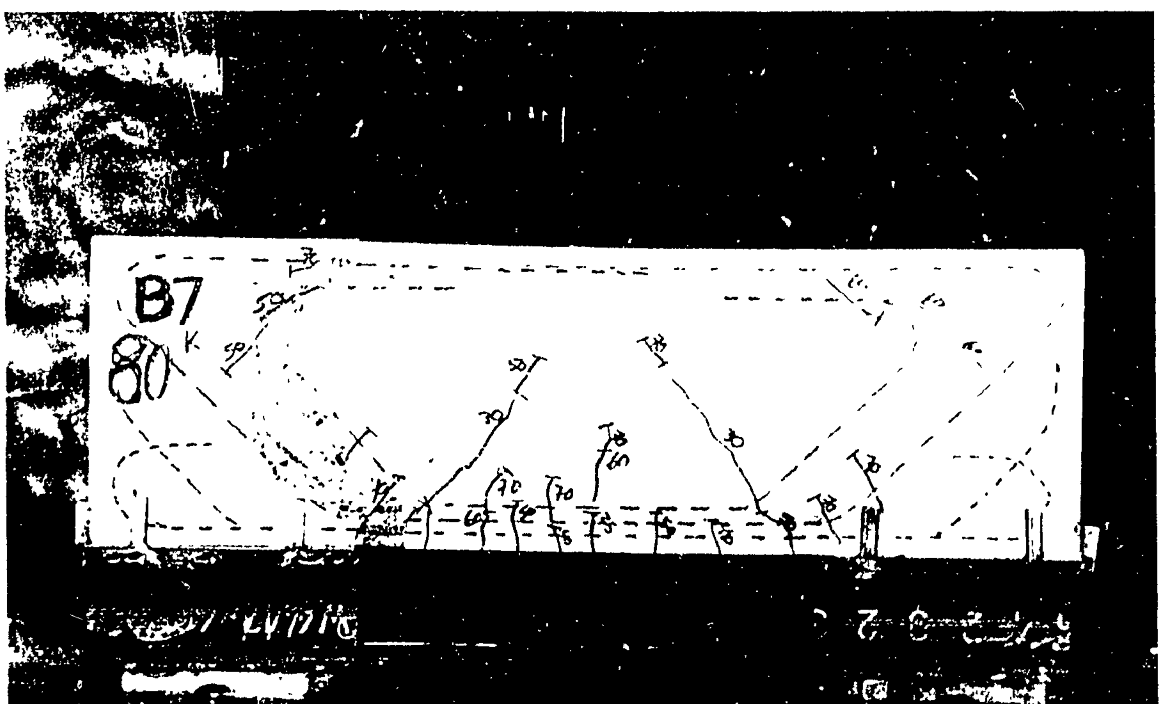


FIGURE A.26 Cracking pattern for sample B7 (failed at 40.0 kips or 177.9 kN)

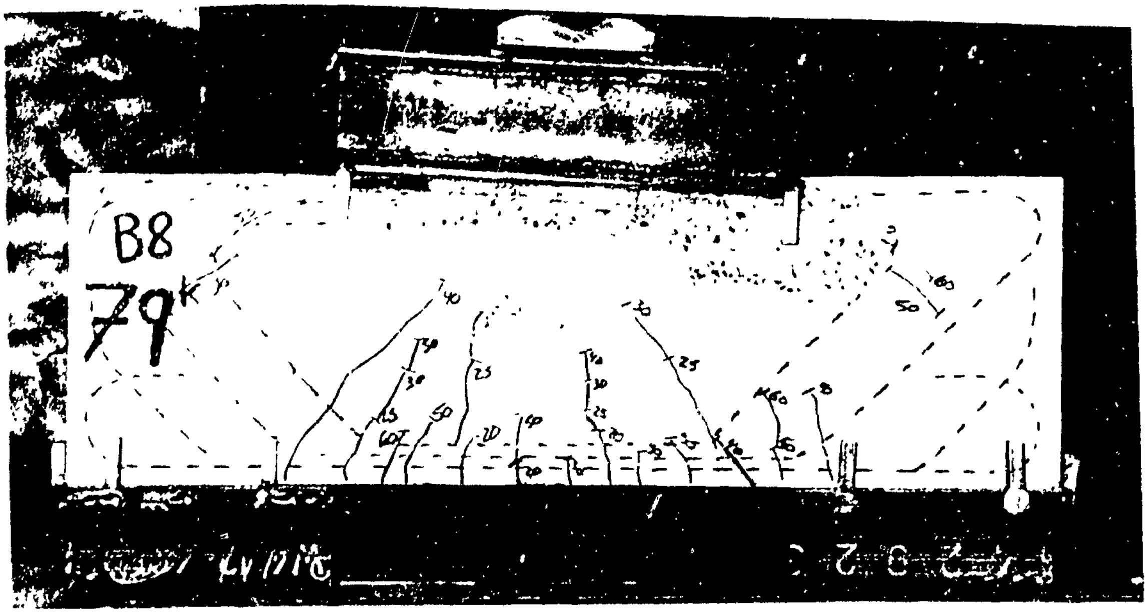


FIGURE A.27 Cracking pattern for sample B8 (failed at 46.6 kips or 207.3 kN)

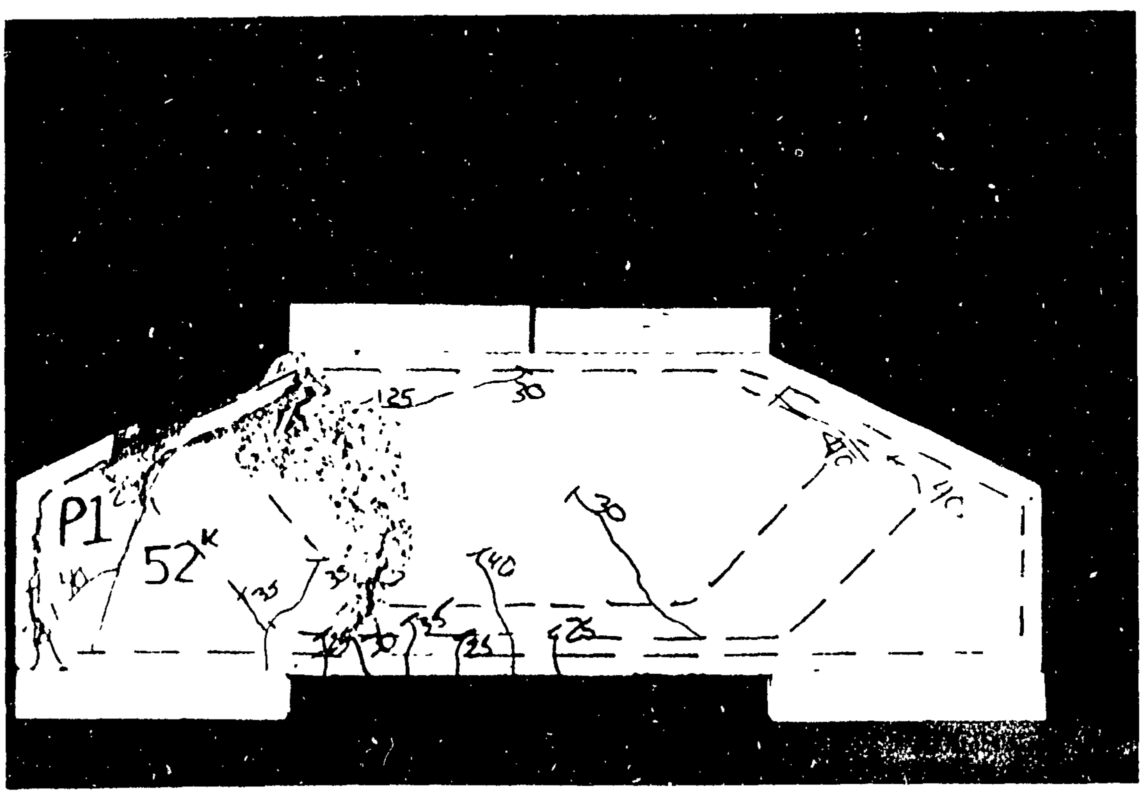


FIGURE A.28 Cracking pattern for sample P1 (failed at 26.0 kips or 115.6 kN)

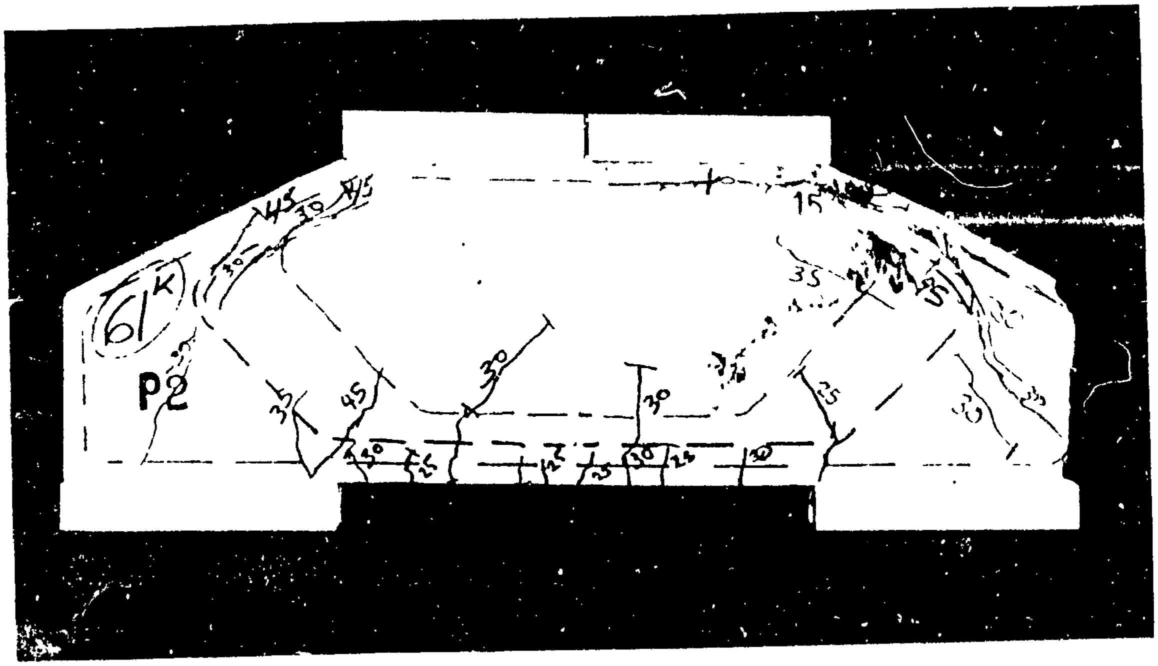


FIGURE A.29 Cracking pattern for sample P2 (failed at 30.5 kips or 135.6 kN)

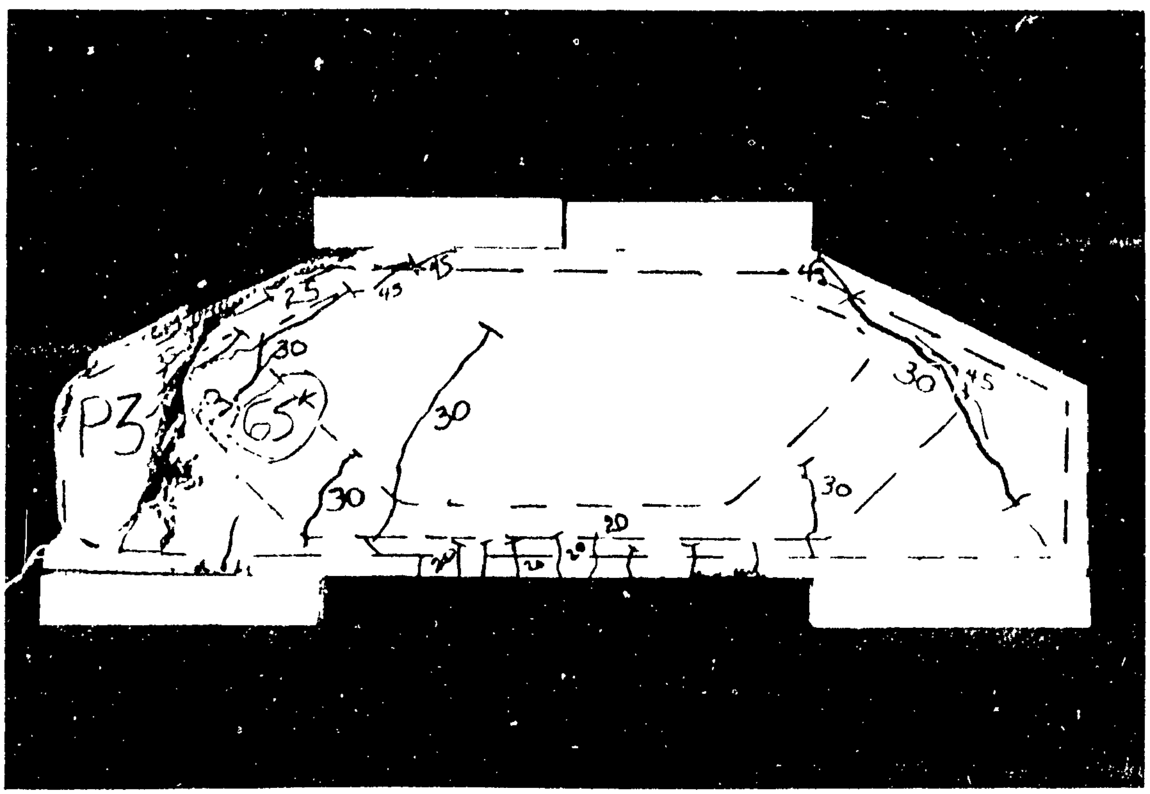


FIGURE A.30 Cracking pattern for sample P3 (failed at 32.5 kips or 144.6 kN)

APPENDIX B- Stress-Strain Relationship

TABLE B.1 Stress-strain $\times 10^{-6}$ data for sample R3

LOAD	2.5 kips (11.1kN)	5.0 kips (22.2kN)	7.5 kips (33.4kN)	10.0 kips (44.5kN)	12.5 kips (55.6kN)	15.0 kips (66.7kN)	17.5 kips (77.8kN)	20.0 kips (89.0kN)	22.5 kips (100 kN)	25 kips (111 kN)	27.5 kips (122 kN)	28.5 kips (127 kN)
	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi
Strain gauge 1	64.0 [1856]	162.0 [4698]	287.0 [8323]	456.0 [13224]	609.0 [17661]	872.0 [25288]	1119.0 [32451]	1357.0 [39353]	1528.0 [44312]	1841.0 [53389]	2208.0 [64032]	2289.0 [66381]
Strain gauge 2	81.0 [2349]	216.0 [6264]	400.0 [11600]	662.0 [19198]	902.0 [26158]	1185.0 [34365]	1468.0 [42572]	1748.0 [50692]	1946.0 [56434]	2250.0 [65250]	2515.0 [72935]	3385.0 [98165]
Strain gauge 3	23.0 [667]	58.0 [1682]	88.0 [2552]	316.0 [9164]	519.0 [15051]	682.0 [19778]	854.0 [24766]	1021.0 [29609]	1122.0 [32538]	1280.0 [37120]	1454.0 [42166]	1636.0 [47444]
Strain gauge 4	47.0 [1363]	112.0 [3248]	213.0 [6177]	403.0 [11687]	502.0 [14558]	593.0 [17197]	724.0 [20996]	879.0 [25491]	1013.0 [29377]	1162.0 [33698]	1383.0 [40107]	1480.0 [42920]
Strain gauge 5	26.0 [754]	41.0 [1189]	42.0 [1218]	27.0 [783]	87.0 [2523]	148.0 [4292]	215.0 [6235]	317.0 [9193]	403.0 [11687]	541.0 [15689]	679.0 [19691]	908.0 [26332]
Strain gauge 6	27.0 [783]	43.0 [1247]	76.0 [2204]	144.0 [4176]	187.0 [5423]	222.0 [6438]	262.0 [7598]	327.0 [9483]	379.0 [10991]	471.0 [13659]	564.0 [16356]	564.0 [16356]

TABLE B 2 Stress-strain $\times 10^{-6}$ data for sample Q3

LOAD	2 5 kips (11 1kN)	5 0 kips (22 2kN)	7 5 kips (33 4kN)	10 0 kips (44 5kN)	12 5 kips (55 6kN)	15 0 kips (66 7kN)	17 5 kips (77 8kN)	20 0 kips (89 0kN)	22 5 kips (100 kN)	25 kips (111 kN)	27 5 kips (122 kN)	30 kips (133 kN)	32 5 kips (144 kN)
	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi
Strain gauge 1	48 0 [1392]	100 0 [2900]	167 0 [4843]	317 0 [9193]	435 0 [12615]	544 0 [15776]	624 0 [18386]	735 0 [21315]	844 0 [24476]	962 0 [27898]	1080 0 [31320]	1201 0 [34829]	1310 0 [37990]
Strain gauge 2	74 0 [2146]	141 0 [4089]	233 0 [6757]	452 0 [13108]	592 0 [17168]	787 0 [22823]	958 0 [27782]	1148 0 [33292]	1325 0 [38425]	1493.0 [43297]	1655 0 [47995]	1823 0 [52867]	1982 0 [57478]
Strain gauge 3	33 0 [957]	62 0 [1798]	105 0 [3045]	193 0 [5597]	298 0 [8642]	388 0 [11252]	456 0 [13224]	550 0 [15950]	640 0 [18560]	740 0 [21460]	845 0 [24505]	948 0 [27492]	1075 0 [31175]
Strain gauge 4	53 0 [1537]	108.0 [3132]	192 0 [5568]	408 0 [11832]	529 0 [15341]	661 0 [19169]	759 0 [22011]	886 0 [25694]	1013 0 [29377]	1148 0 [33292]	1284 0 [37236]	1452 0 [42108]	1612 0 [46748]
Strain gauge 5	19 0 [551]	34.0 [986]	60.0 [1740]	200 0 [5800]	312.0 [9048]	380 0 [11020]	416 0 [12064]	501.0 [14529]	586 0 [16994]	697 0 [20213]	786 0 [22794]	832 0 [24128]	925 0 [26825]
Strain gauge 6	81 0 [2349]	135 0 [3915]	212 0 [6148]	529.0 [15341]	770 0 [22330]	955 0 [27695]	1050 0 [30450]	1219 0 [35351]	1251 0 [36279]	1320 0 [38280]	1375 0 [39875]	1414 0 [41006]	1367 0 [39643]

TABLE B.3 Stress-strain$\times 10^{-6}$ data for sample T1

LOAD	1.25 kips (5.6 kN)	2.5 kips (11.1 kN)	3.75 kips (16.8kN)	5.0 kips (22.4kN)	6.25 kips (27.8kN)	7.5 kips (33.4kN)	8.75 kips (38.9kN)	10.0 kips (44.5kN)	11.2 kips (50.0kN)	12.5 kips (55.6kN)	13.7 kips (61.2kN)
	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi
Strain gauge 1	1421.0 [9.8]	2001.0 [13.8]	3654.0 [25.2]	5655.0 [39.0]	8120.0 [56.0]	10875.0 [75.0]	14239.0 [98.2]	16095.0 [111.0]	20300.0 [140.0]	22591.0 [155.8]	33901.0 [233.8]
Strain gauge 2	957.0 [6.6]	2001.0 [13.8]	3393.0 [23.4]	5336.0 [36.8]	6670.0 [46.0]	8903.0 [61.4]	11310.0 [78.0]	13804.0 [95.2]	15660.0 [108.0]	19836.0 [136.0]	26100.0 [180.0]
Strain gauge 3	783.0 [5.4]	1798.0 [12.4]	2755.0 [19.0]	4002.0 [27.6]	4930.0 [34.0]	6380.0 [44.0]	8381.0 [57.8]	10469.0 [72.2]	12412.0 [85.6]	15863.0 [109.4]	19865.0 [137.0]
Strain gauge 4	1044.0 [7.2]	2030.0 [14.0]	3132.0 [21.6]	6322.0 [43.6]	7366.0 [50.8]	8903.0 [61.4]	10730.0 [74.0]	12557.0 [86.6]	14471.0 [99.8]	16835.0 [113.0]	15979.0 [110.2]
Strain gauge 5	696.0 [4.8]	1102.0 [7.6]	1624.0 [11.2]	3132.0 [21.6]	5220.0 [36.0]	7134.0 [49.2]	10208.0 [70.4]	13050.0 [90.0]	16971.0 [115.8]	20416.0 [140.8]	26825.0 [185.0]
Strain gauge 6	493.0 [3.4]	899.0 [6.2]	1595.0 [11.0]	3277.0 [22.6]	6090.0 [42.0]	7888.0 [54.4]	10817.0 [74.6]	12818.0 [88.4]	14674.0 [101.2]	16356.0 [112.8]	16820.0 [116.0]

TABLE B 4 Stress-strain$\times 10^{-6}$ data for sample T2

LOAD	1 25 kips (5 6 kN)	2 5 kips (11 1 kN)	3 75 kips (16 8kN)	5 0 kips (22 4kN)	6 25 kips (27 8kN)	7 5 kips (33 4kN)	8 75 kips (38 9kN)	10 0 kips (44 5kN)	11 2 kips (50 0kN)	12 5 kips (55 6kN)	13 7 kips (61 2kN)	15 5 kips (68 9kN)	16 2 kips (72 3kN)
	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi
Strain gauge 1	1682 0 [11 6]	3277 0 [22 6]	8004 0 [55.2]	11310 0 [78.0]	17661 0 [121 8]	22243 0 [153.4]	27666 0 [190 8]	31755 0 [219 0]	38048 0 [262 4]	43297 0 [298 6]	49184 0 [339 2]	55071 0 [379 8]	64728 0 [446 4]
Strain gauge 2	1015 0 [7 0]	2900 0 [20 0]	3799 0 [26 2]	8787.0 [60 6]	12151 0 [83.8]	15051 0 [103 8]	17226 0 [118 8]	21257 0 [146 6]	23837 0 [164 4]	28565 0 [197 0]	32422 0 [223.6]	37990 0 [262 0]	42804 0 [295 2]
Strain gauge 3	348 0 [2 4]	1044 0 [7 2]	1421 0 [9 8]	2465.0 [17.0]	3973 0 [27.4]	7946 0 [54 8]	10150 0 [70.0]	13224 0 [91.2]	15022 0 [103 6]	17226 0 [118 8]	19082 0 [131 6]	21228 0 [146 4]	22852 0 [157 6]
Strain gauge 4	N.A.	N.A.	N.A	N.A.	N A.	N.A.	N A	N A.	N.A.	N A	N A.	N.A.	N A
Strain gauge 5	232 0 [1.6]	580.0 [4 0]	725 0 [5 0]	696.0 [4.8]	261 0 [1.8]	1682 0 [11.6]	2697 0 [18 6]	3509 0 [24 2]	4408 0 [30.4]	5104.0 [35.2]	5829.0 [40.2]	6757 0 [46 6]	8497 0 [58 6]
Strain gauge 6	116.0 [0.8]	87.0 [0 6]	145 0 [1.0]	464.0 [3 2]	899.0 [6.2]	1885.0 [13.0]	2204 0 [15 2]	2755 0 [19 0]	3306 0 [22 8]	3480.0 [24 0]	3683.0 [25.4]	4002.0 [27 6]	3944 0 [27 2]

TABLE B 5 Stress-strain$\times 10^{-6}$ data for sample P3

LOAD	2 5 kips (11 1kN)	5 0 kips (22 2kN)	7.5 kips (33 4kN)	10 0 kips (44 5kN)	12 5 kips (55 6kN)	15 0 kips (66 7kN)	17.5 kips (77.8kN)	20.0 kips (89 0kN)	22 5 kips (100 kN)	25 kips (111 kN)	27.5 kips (122 kN)	30 kips (133 kN)	32 5 kips (144 kN)
	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi	strain [stress] psi
Strain gauge 1	51 0 [1479 0]	114 0 [3306 0]	270 0 [7830 0]	405 0 [11745]	522 0 [15138]	754 0 [21866]	876 0 [25404]	1030 0 [29870]	1160 0 [33640]	1310 0 [37990]	1446.0 [41934]	1600 0 [46400]	1767 0 [51243]
Strain gauge 2	51 0 [1479 0]	124 0 [3596 0]	232 0 [6728 0]	338.0 [9802 0]	450 0 [13050]	713 0 [20677]	845 0 [24505]	998 0 [28942]	1137 0 [32973]	1293 0 [37497]	1448 0 [41992]	1618 0 [46922]	1798 0 [52142]
Strain gauge 3	37 0 [1073 0]	88.0 [2552 0]	172 0 [4988 0]	253 0 [7337 0]	367 0 [10643]	452 0 [13108]	507 0 [14703]	578 0 [16762]	648 0 [18792]	723 0 [20967]	803 0 [23287]	889 0 [25781]	979 0 [28391]
Strain gauge 4	19 0 [551 0]	38 0 [1102 0]	101 0 [2929 0]	164 0 [4756 0]	235 0 [6815 0]	280 0 [8120 0]	323 0 [9367 0]	377 0 [10933]	439 0 [12731]	504 0 [14616]	572 0 [16588]	645 0 [18705]	713 0 [20677]
Strain gauge 5	38 0 [1102 0]	85 0 [2465 0]	181 0 [5249 0]	267 0 [7743 0]	358 0 [10382]	376 0 [10904]	410 0 [11890]	459 0 [13311]	513 0 [14877]	564 0 [16356]	625 0 [18125]	680 0 [19720]	722 0 [20938]
Strain gauge 6	28 0 [812 0]	55 0 [1595 0]	122 0 [3538 0]	188 0 [5452 0]	275 0 [7975 0]	295 0 [8555 0]	333 0 [9657 0]	366 0 [10614]	409 0 [11861]	451 0 [13079]	500 0 [14500]	547 0 [15863]	--------

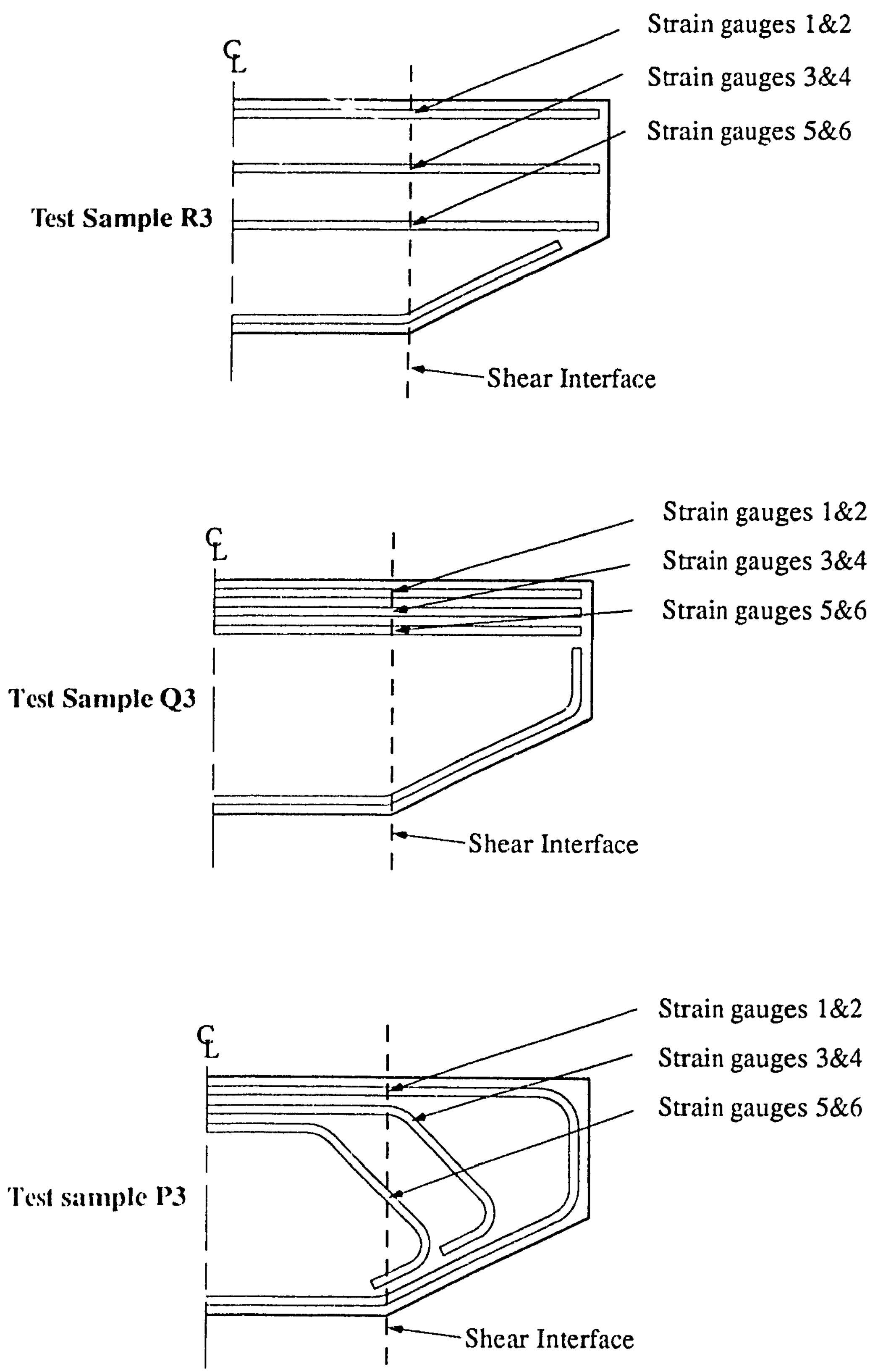


FIGURE B.1 Location of strain gauges on samples R3, Q3 and P3

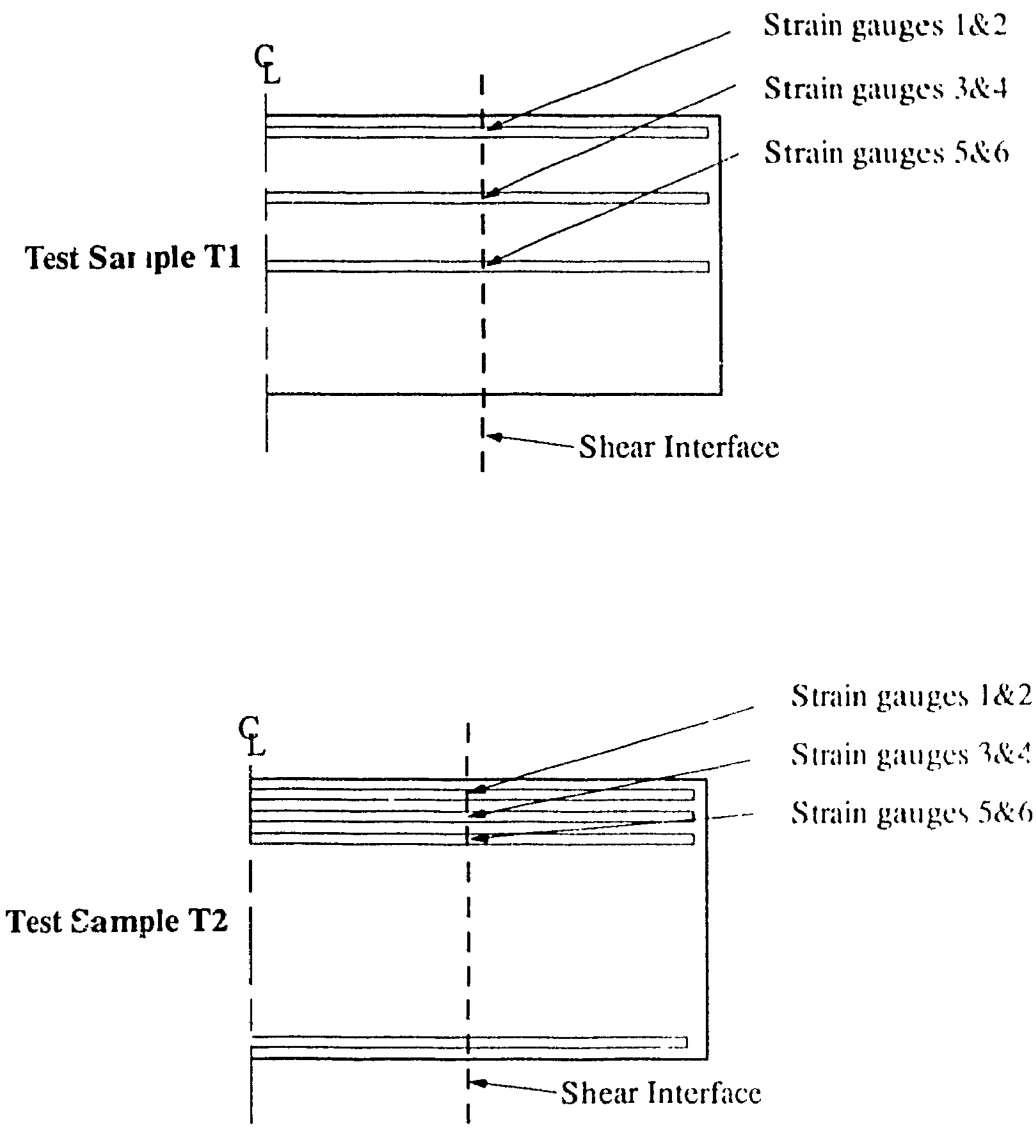


FIGURE B.2 Location of strain gauges on samples T1 and T2

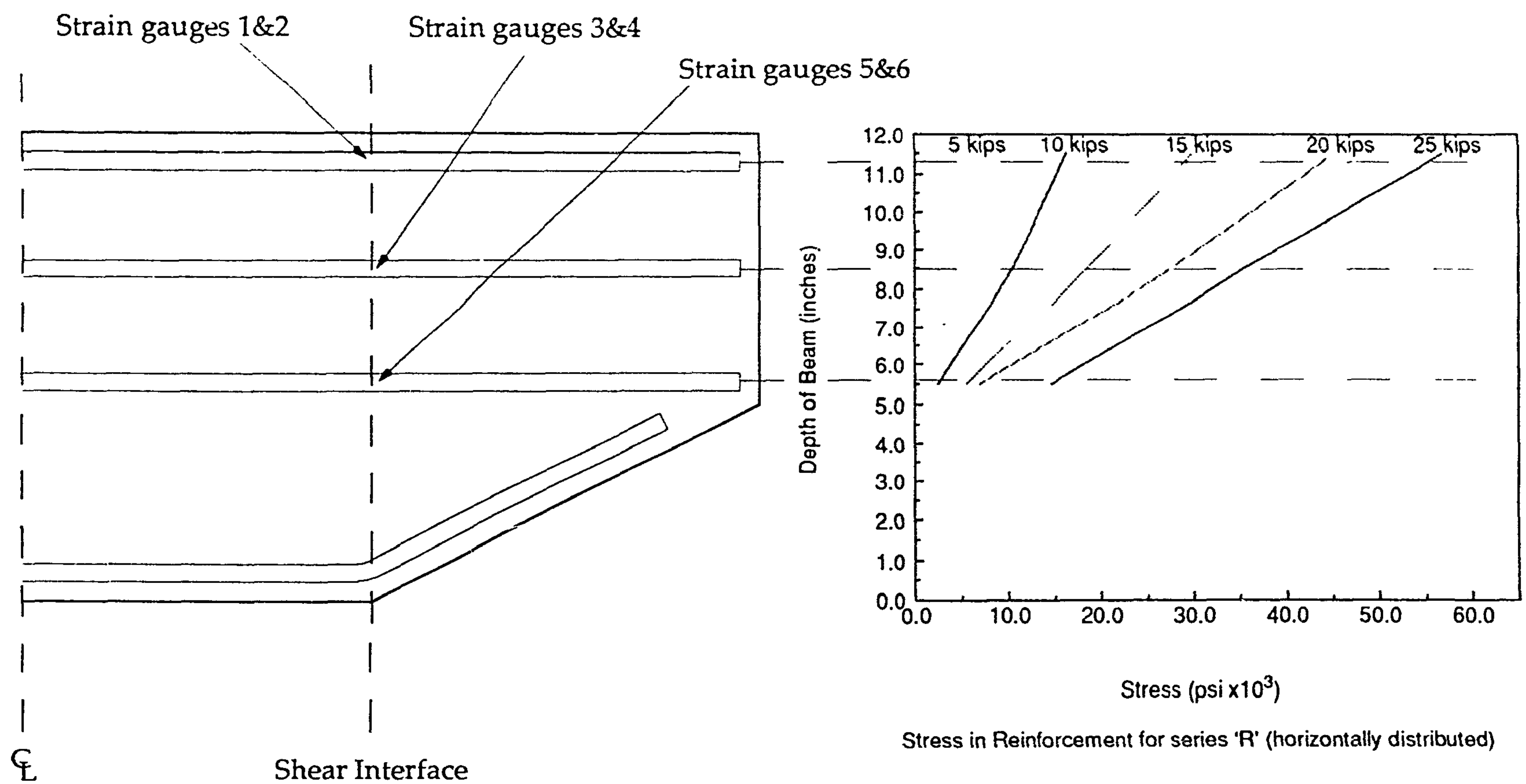


FIGURE B.3 Stress in reinforcement for sample R3

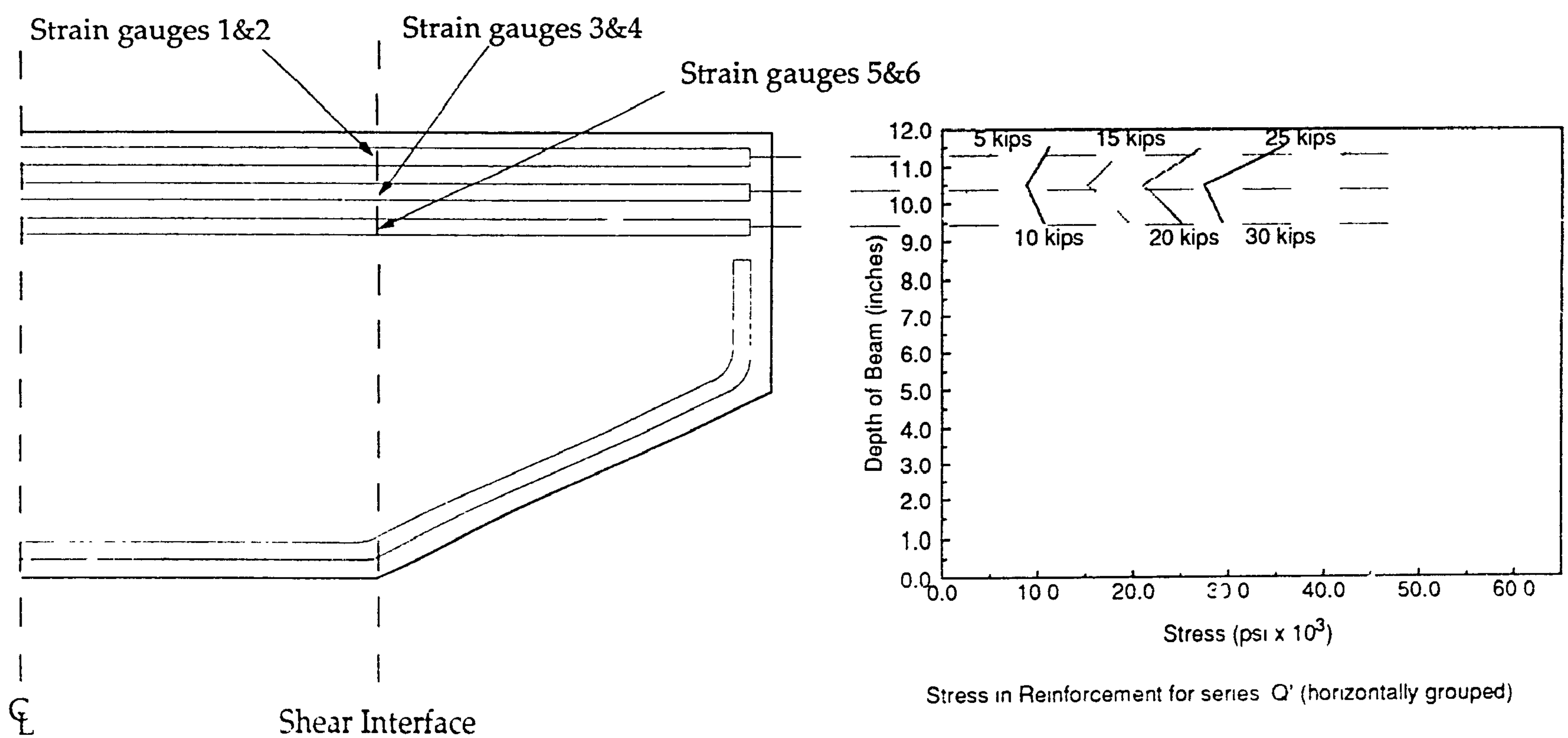


FIGURE B.4 Stress in reinforcement for sample Q3

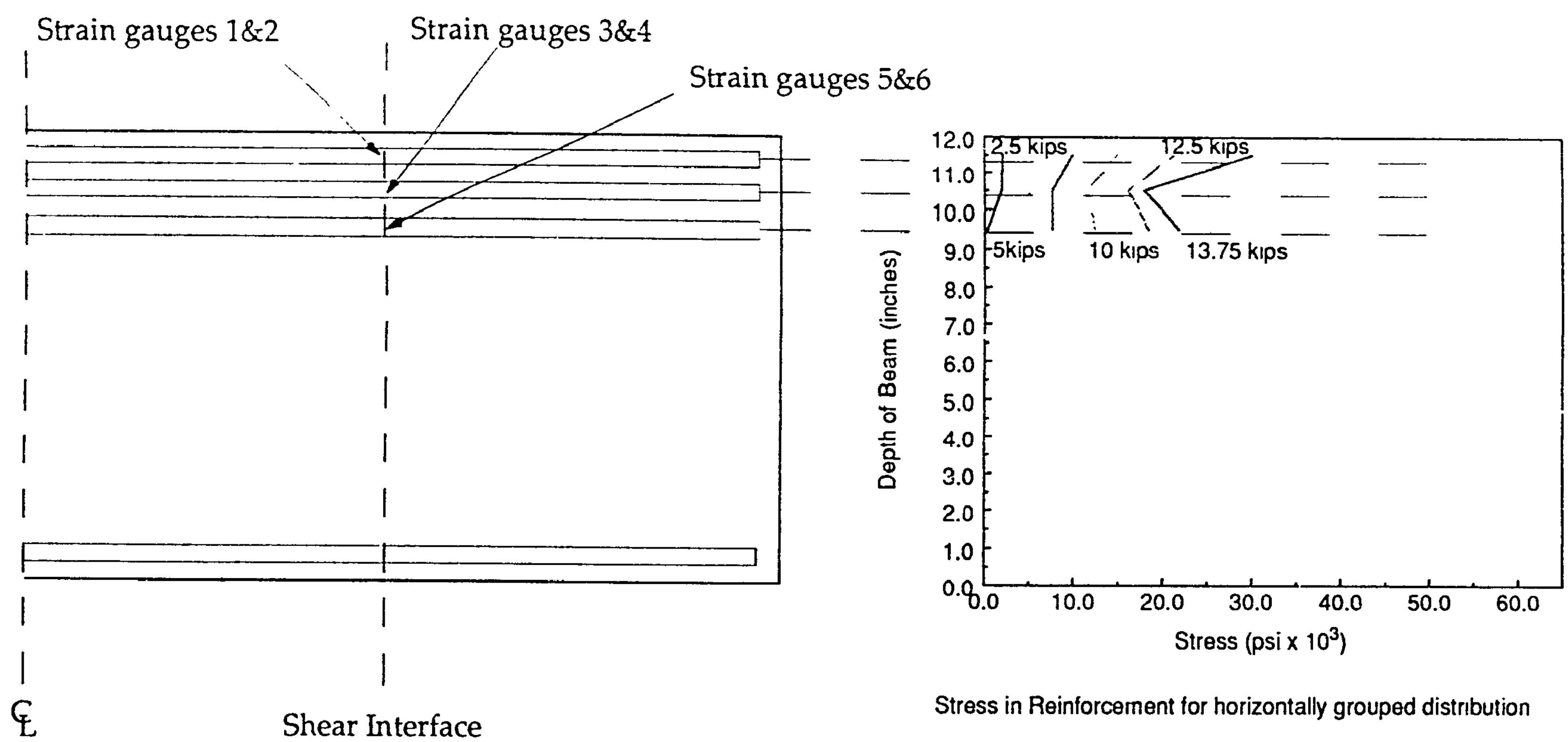


FIGURE B.5 Stress in reinforcement for sample T1

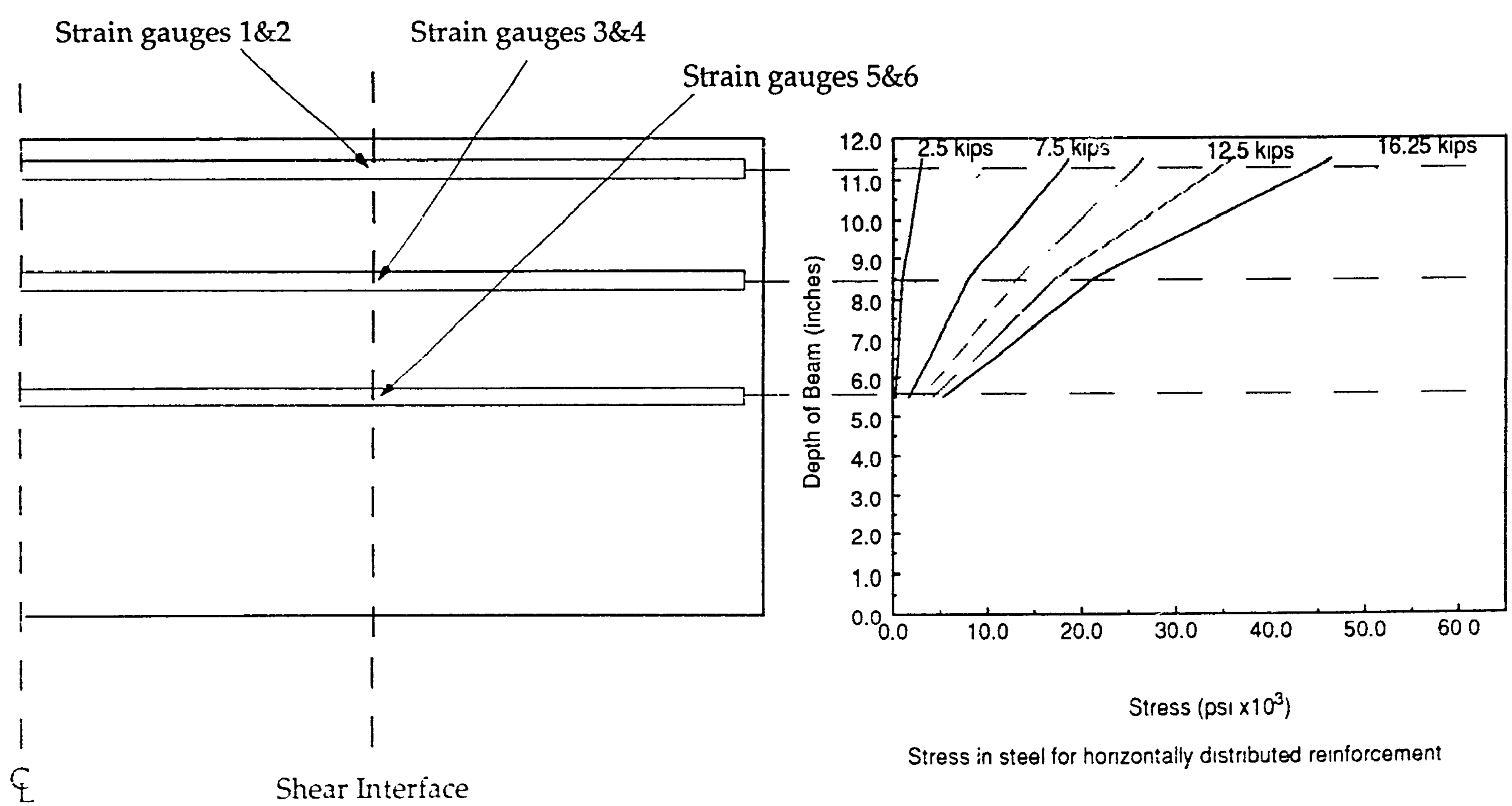


FIGURE B.6 Stress in reinforcement for sample T2

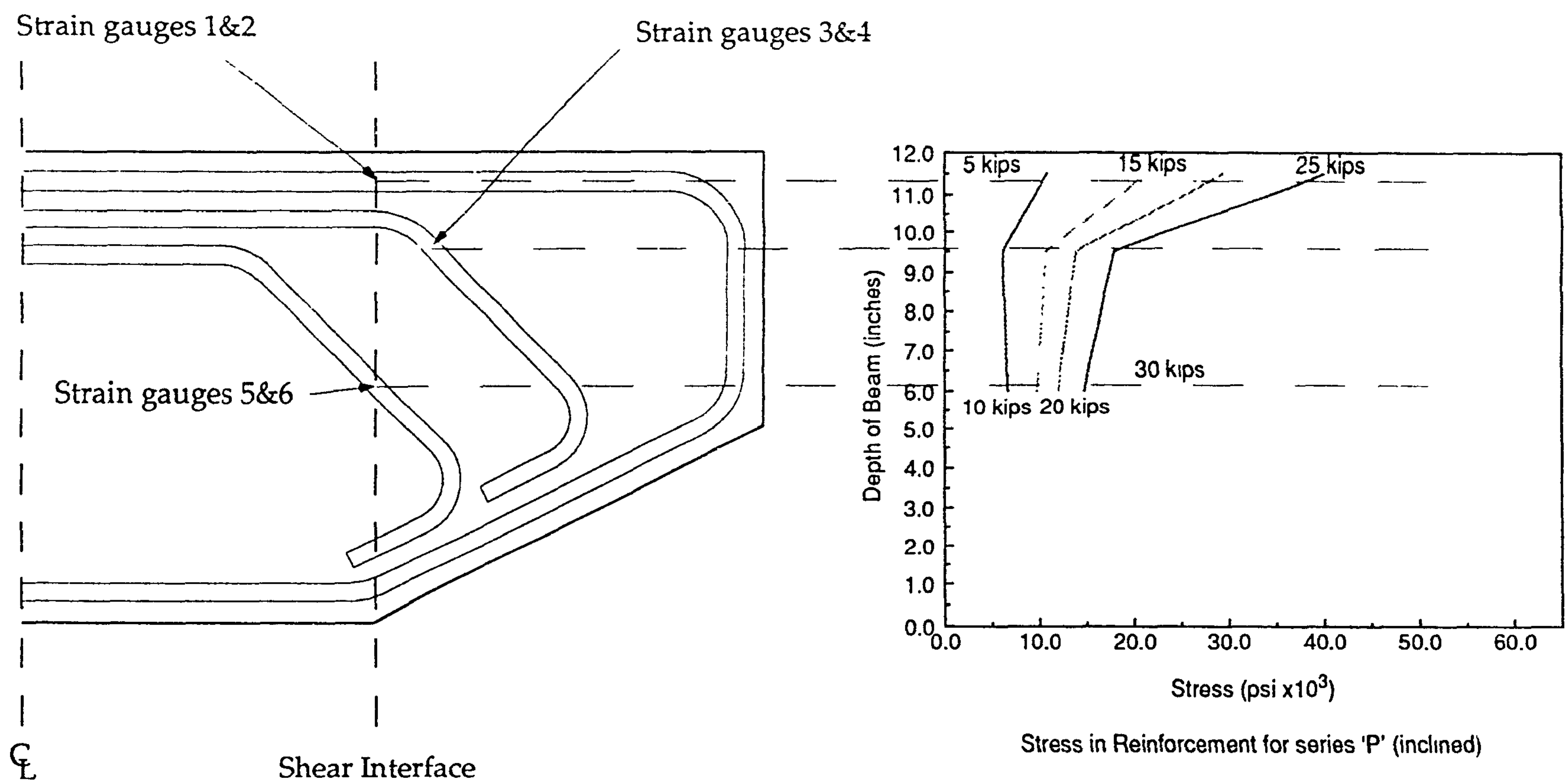


FIGURE B.7 Stress in reinforcement for sample P3

APPENDIX C- Design Examples

C.1 Design of a Corbel

As shown in the figure, a corbel projecting from a 250mm x 250mm column supports a precast beam. The corbel is subjected to a vertical dead load of 80 kN and a vertical specified live load of 110 kN. Due to beam creep and shrinkage a horizontal force of 35 kN will develope. Assume a concrete compressive strength of 35 MPa and a steel yield strength of 400 MPa. Use only horizontal reinforcement.

Design Procedure

1. Factor loads: $V = 1.25D + 1.5L = 1.25(80) + 1.5(110) = 265\text{kN}$

$H = 1.25D = 1.25(35) = 44\text{ kN}$

2. Determine required steel area:

For the horizontal load:

$$A_{hf} = \frac{H}{0.85f_y} = \frac{44\times10^3}{0.85\times400} = 129.4\text{mm}^2$$

For the vertical load:

$$A_{hf} = \frac{V}{0.85f_y} = \frac{265\times10^3}{0.85\times400} = 779.4\text{mm}^2$$

Total horizontal steel required is 908.8 mm^2.

From equation [36] determine the required amount of concrete.

$$A_{cv} = bh = \frac{V}{0.424\phi_c f'_c} = \frac{265\times10^3}{0.424\times0.6\times35} = 29762\text{mm}^2$$

Determine the ratio of reinforcement and compare with the minimum requirements as defined by equation [32].

$$\rho_{hf} = \frac{A_{hf}}{bh} = \frac{779.4}{29762} = 0.02618$$

The minimum required reinforcement ratio is:

$$\rho_{hf} \geq \frac{0.5\sqrt{f'_c}}{f_y - 14}$$

$$0.02618 \geq \frac{0.5\sqrt{35}}{400 - 14} = 0.0076 \qquad \text{ok}$$

Assume a concrete corbel 175mm wide, 250 mm deep and 150 mm long. The ratio of a/h=0.6 <1 therefore acceptable.The chosen section has a concrete area of 43750mm^2 which is greater then the required area. For a steel area of 908.8mm^2, use two No. 25 bars, for a total area of 1000 mm^2.

Check shear capacity

The concrete capacity is V_{uc}=0.424ϕ_cf'$_c$bh=0.424x0.6x35x43750=389.5 kN

The steel capacity is V_{us}=0.85$A_{hf}f_y$ =0.85x1000x400= 340 kN of which 44 kN is required for the horizontal load. V_{us}<V_{uc}, which ensures a steel failure.

Check moment capacity

To check the moment capacity it is required to determine the lever arm between the horizontal component of concrete capacity and the reinforcement capacity.

The total tension capacity = $\phi_s A_{hf} f_y$ = 0.85 x 1000 x 400 =340 kN

$$a = \frac{\phi_s A_{hf} f_y}{0.85^2 \phi_c f'_c b} = \frac{340}{1896.5} = 180\text{mm}$$

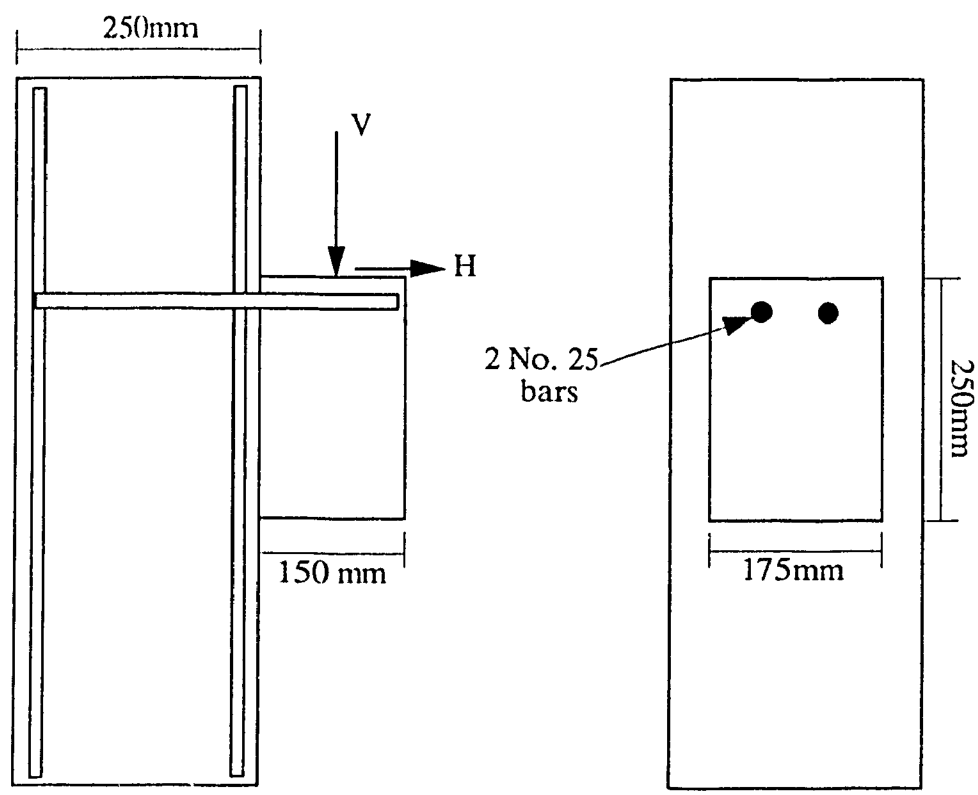
Assume a cover of 25.4 mm, the lever arm z=250-90-25.4=134.6mm

Moment capacity= Tz = (340)(134.6) =45.7 kNm.

Total moment at the shear interface =(265)(75) + (44)(25) =20.9 kNm.

The moment capacity exceeds the actual moment therefore the design is adequate.

A more economical may be achieved by selecting a different concrete area or smaller reinforcement. The final design is shown in the following figure.



Note: The development length was not considered, it may be necessary to use a "hook"
Cover of 25.4 mm was assumed.

C.2 Design of a deep beam

The deep beam shown is to carry a dead load of 80 kips at its mid span. Neglecting the weight of the beam, design the required reinforcement.

Given: f'_c=3000 psi, f_y=40000 psi

<u>Design</u>

1. Check ratio of $a/h<1$

   shear span a=10 inches, height =15 inches, $a/h=0.6<1$ therefore okay.

2) Factor loads as seen from the support.

$$V = \frac{1.4D + 1.7L}{\phi_{shear}} = \frac{1.4 \times 40}{0.85} = 66\text{kips}$$

3) Concrete shear area=10" x 15" =150 in^2

4) Calculate $K = \frac{V}{\text{Area}} = \frac{66000}{150} = 440\text{psi}$

5) From Table 6.2, for f'_c=3000 psi, $\rho=0.011$. ($\rho_{min}<\rho<\rho_{max}$)

7) Area of steel, $Ahf=(A_{cv})(\rho) = (150)(0.011) = 1.65\ in^2$.

   Use 3 # 7 bars for a total area of 1.8 in^2

8) Check moment capacity

   T=(1.8)(40000psi) =72 kips

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{72000}{0.85 \times 3000 \times 10} = 2.82\text{in}$$

Assume a one (1) inch cover.

Lever arm z=15-2.82/2-1=12.6 in

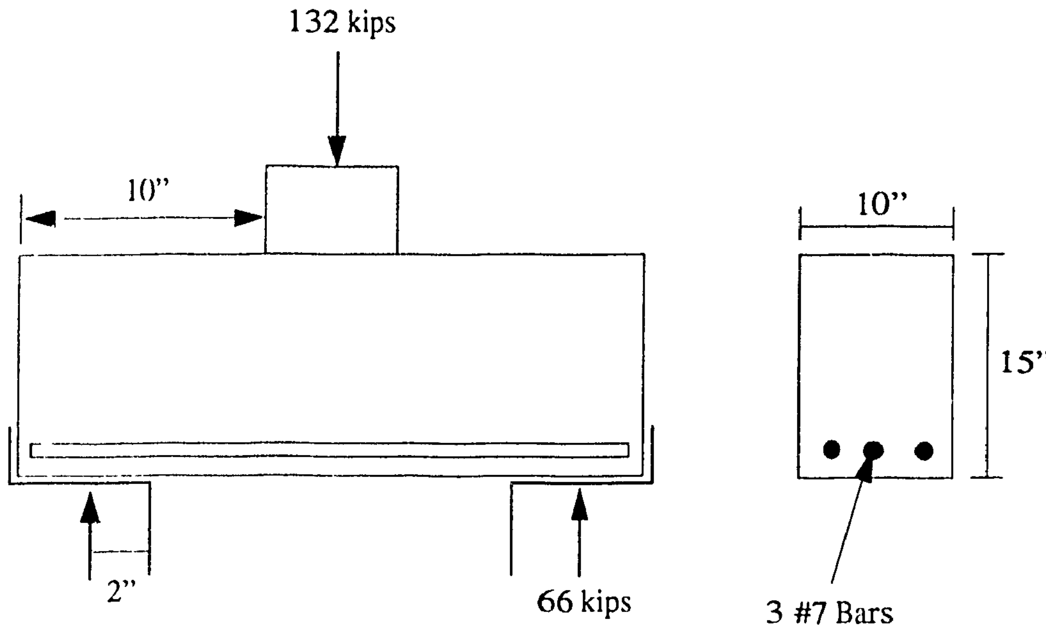
Since $\rho<\rho_{max}$, the tension capacity is less then the horizontal component of the concrete capacity.

Moment capacity =Tz = (72000)(12.6) =75.6 ftkip

Actual moment = (66)(8/12) = 44 ftkip

Since the moment capacity > actual moment therefore okay

The following diagram illustrates the final design.



Note: The development length was not considered, it may be necessary to use a "hook"
Cover of 25.4 mm was assumed.